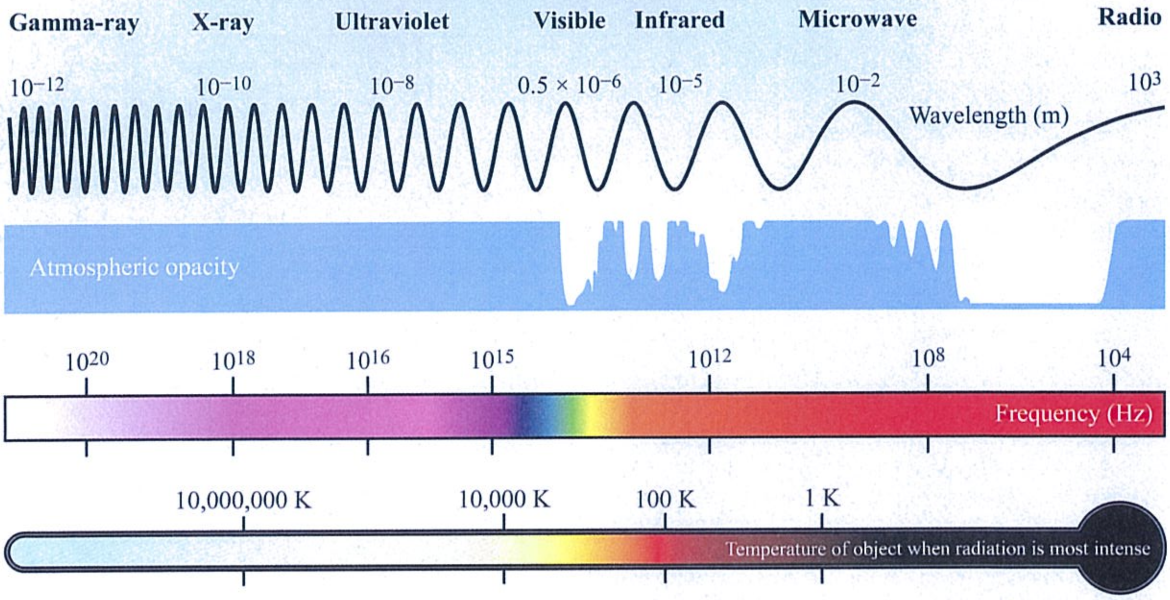
CHAPTER

2

Electromagnetic Wave Propagation and Reflection



Electromagnetic spectrum

CONTENTS

Overview

To understand how active and passive microwave sensors operate and how the ***electromagnetic*** (EM) quantities they measure are transformed into biological, hydrological, and geophysical information about Earth's surface and atmosphere, it is imperative that we have a solid understanding of EM theory and *how EM waves interact with material media.* The interaction may involve ***scattering***, ***absorption***, ***transmission***, and ***emission***, or combinations thereof. Accordingly, this chapter provides a review of EM waves and their properties, EM propagation in lossy and lossless media, and wave reflection and transmission at planar boundaries. This review will allow us in future chapters to develop models for scattering by boundaries with rough surfaces and by volumes containing particles (such as water droplets in clouds, rain and snow, and leaves and needles in vegetation-covered canopies).

In a parallel treatment, Chapter 6 establishes the connection between the EM energy naturally emitted by a surface or medium (such as the atmosphere) and the dielectric and geometrical properties of that surface or medium. The scattered EM energy measured by a radar and the emitted energy measured by a radiometer are such that they depend not only on the properties of the scene, but also on the properties of the microwave sensor, namely its EM wavelength, observation direction relative to the scene, and the polarization configuration of its antenna(s). We will keep these ***sensor parameters*** in mind throughout our forthcoming discussions.

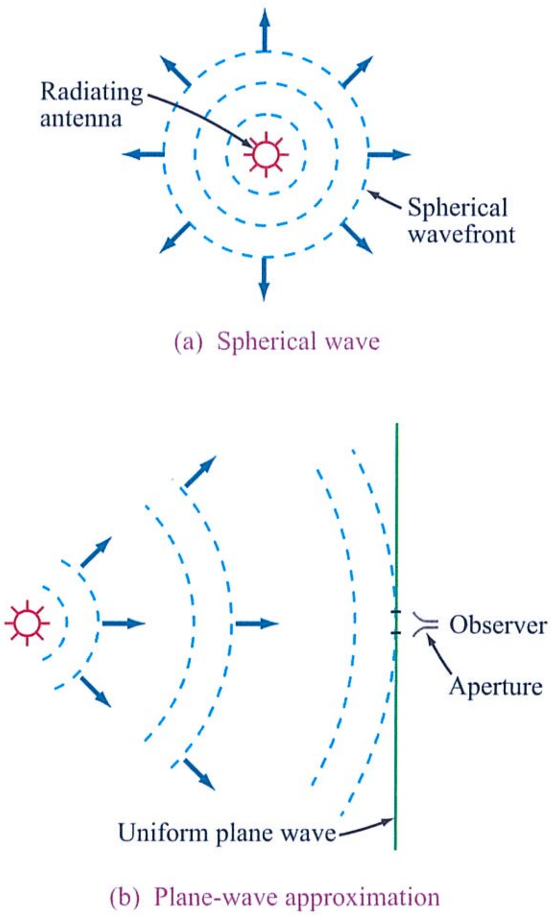


Figure 2-1: Waves radiated by an EM source, such as a lightbulb or an antenna, have spherical wavefronts, as in (a); to a distant observer, however, the wavefront across the observer's aperture appears approximately planar, as in (b).

2-1 EM Plane Waves

A time-varying electric field induces a magnetic field and, conversely, a time-varying magnetic field induces an electric field. This cyclic pattern often results in electromagnetic waves propagating through free space and in material media. When a wave propagates through a homogeneous medium without interacting with obstacles or material interfaces, it is said to be ***unbounded***. Light waves emitted by the sun and radio transmissions by antennas are good examples. Unbounded waves may propagate in both lossless and lossy media. Waves propagating in a ***lossless medium*** (e.g., air and perfect dielectrics) do not attenuate. When propagating in a ***lossy medium*** (material with nonzero conductivity, such as water), part of the power carried by an EM wave gets converted into heat.

A wave produced by a localized source, such as an antenna, expands outwardly in the form of a ***spherical wave***, as depicted in Fig. 2-1(a). Even though an antenna may radiate more energy along some directions than along others, the spherical wave travels at the same speed in all directions. To an observer very far away from the source, however, the ***wavefront*** of the spherical wave appears approximately *planar*, as if it were part of a ***uniform plane wave*** with identical properties at all points in the plane tangent to the wavefront [Fig. 2-1(b)]. Plane waves are easily described using a Cartesian coordinate system, which is mathematically easier to work with than the spherical coordinate system needed to describe spherical waves.

2-1.1 Constitutive Parameters

Regardless of its material composition, a small differential volume is characterized by four electromagnetic ***constitutive parameters***:

- $\varepsilon'\varepsilon_0$ = ***electrical permittivity*** (F/m),†
- μ = ***magnetic permeability*** (H/m),
- ρ_v = ***volume charge density*** (C/m^3),
- σ = ***conductivity*** (S/m)

where $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m is the permittivity of free space and ε' is the ***permittivity of the material* relative relative *to that of free space***.

> ▶ We denote the ***relative permittivity*** by the symbol ε', instead of ε, so we may use the latter in future sections to represent the complex dielectric constant of the material under sinusoidal time-varying conditions. ◀

Examples of constitutive parameters:

(a) Free space:

$\varepsilon' = 1$

$\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m

$\rho_v = 0$

$\sigma = 0$

(b) Pure dielectric:

ε': medium-specific

$\mu = \mu_0$ (all materials except ferromagnetic)

$\rho_v = 0$

$\sigma = 0$

(c) Conducting medium:

ε': medium-specific

$\mu = \mu_0$ (except ferromagnetic)

ρ_v may or may not be zero

$\sigma \neq 0$

As we shall see shortly, an EM wave can propagate through a dielectric medium (including free space) with no loss of energy (zero attenuation). In contrast, a conducting medium absorbs part of the energy carried by an EM wave traveling through it, thereby attenuating it.

2-1.2 Maxwell's Equations

In a homogeneous, isotropic medium, the differential form of Maxwell's equations is given by

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon'\varepsilon_0} \quad \textbf{(Gauss's law)}, \tag{2.1a}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \quad \textbf{(Faraday's law)}, \tag{2.1b}$$

$$\nabla \cdot \mathbf{H} = 0 \quad \textbf{(Gauss's law for magnetism)}, \tag{2.1c}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon'\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad \textbf{(Ampère's law)}, \tag{2.1d}$$

where **E** is the ***electric field intensity*** (in V/m), **H** is the ***magnetic field intensity*** (in A/m), and **J** is the ***current density*** (in A/m^2) flowing through the medium.

Time-varying electric and magnetic fields (**E** and **H**) and their sources (the charge density ρ_v and current density **J**) generally depend on the spatial coordinates (x,y,z) and the time variable t. However, if their time variation is sinusoidal with angular frequency ω, then these quantities can be represented by a phasor that depends on (x,y,z) only. The vector phasor $\mathbf{E}(x,y,z)$ and the instantaneous field $\mathbf{E}(x,y,z;t)$ it describes are related as

$$\mathbf{E}(x,y,z;t) = \mathfrak{Re}\left[\mathbf{E}(x,y,z)\, e^{j\omega t}\right], \tag{2.2}$$

†Abbreviations for units of physical quantities are given in Appendix A.

where $j = \sqrt{-1}$. Similar definitions apply to ρ_v and $\mathbf{J}$. For time-harmonic quantities, differentiation in the time domain corresponds to multiplication by $j\omega$ in the phasor domain. Maxwell's equations (Eq. (2.1)) assume the following form in the phasor domain:

$$\nabla \cdot \mathbf{E} = \rho_v / \varepsilon' \varepsilon_0, \tag{2.3a}$$

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}, \tag{2.3b}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{2.3c}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\varepsilon'\varepsilon_0\mathbf{E}. \tag{2.3d}$$

2-1.3 Complex Permittivity

In a medium with conductivity σ, the conduction current density $\mathbf{J}$ is related to $\mathbf{E}$ by $\mathbf{J} = \sigma\mathbf{E}$. Assuming no other current flows in the medium, Eq. (2.3d) may be written as

$$\begin{aligned}\nabla \times \mathbf{H} &= \mathbf{J} + j\omega\varepsilon'\varepsilon_0\mathbf{E} \\ &= (\sigma + j\omega\varepsilon'\varepsilon_0)\mathbf{E} = j\omega\varepsilon_0\left(\varepsilon' - j\frac{\sigma}{\omega\varepsilon_0}\right)\mathbf{E}.\end{aligned} \tag{2.4}$$

By defining the ***complex dielectric constant*** ε as

$$\varepsilon = \varepsilon' - j\frac{\sigma}{\omega\varepsilon_0}, \tag{2.5}$$

Eq. (2.4) can be rewritten as

$$\nabla \times \mathbf{H} = j\omega\varepsilon\varepsilon_0\mathbf{E}. \tag{2.6}$$

Taking the divergence of both sides of Eq. (2.6), and recalling that the divergence of the curl of any vector field vanishes (i.e., $\nabla \cdot \nabla \times \mathbf{H} = 0$), it follows that $\nabla \cdot (j\omega\varepsilon\varepsilon_0\mathbf{E}) = 0$, or $\nabla \cdot \mathbf{E} = 0$. Comparing this with Eq. (2.3a) implies that $\rho_v = 0$. This result is a consequence of the assumption that the only current that may exist in the medium is due to conduction. Upon replacing Eq. (2.3d) with Eq. (2.6) and setting $\rho_v = 0$ in Eq. (2.3a), Maxwell's equations become

$$\nabla \cdot \mathbf{E} = 0, \tag{2.7a}$$

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}, \tag{2.7b}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{2.7c}$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\varepsilon_0\mathbf{E}. \tag{2.7d}$$

The complex dielectric constant ε given by Eq. (2.5) is often written in terms of a real part ε' and an imaginary part ε''. Thus,

$$\varepsilon = \varepsilon' - j\frac{\sigma}{\omega\varepsilon_0} = \varepsilon' - j\varepsilon'', \tag{2.8}$$

with

$$\varepsilon'' = \frac{\sigma}{\omega\varepsilon_0}. \tag{2.9}$$

Since ε'' is associated with heat loss in the material, it is called the ***dielectric loss factor*** of the material. For a lossless medium with $\sigma = 0$, it follows that $\varepsilon'' = 0$ and $\varepsilon = \varepsilon'$.

2-1.4 Wave Equations

To derive wave equations for $\mathbf{E}$ and $\mathbf{H}$ and solve them to obtain explicit expressions for $\mathbf{E}$ and $\mathbf{H}$ as a function of the spatial variables (x, y, z), we take the curl of both sides of Eq. (2.7b) and then use appropriate substitutions to obtain the ***homogeneous wave equation*** for $\mathbf{E}$, namely

$$\nabla^2\mathbf{E} + \omega^2\mu\varepsilon\varepsilon_0\mathbf{E} = 0. \tag{2.10}$$

In Cartesian coordinates the Laplacian of $\mathbf{E}$ is given by

$$\nabla^2\mathbf{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\mathbf{E}. \tag{2.11}$$

By defining the ***propagation constant*** γ as

$$\gamma^2 = -\omega^2\mu\varepsilon\varepsilon_0, \tag{2.12}$$

Eq. (2.10) can be written as

$$\nabla^2\mathbf{E} - \gamma^2\mathbf{E} = 0. \tag{2.13}$$

In deriving Eq. (2.13), we started by taking the curl of both sides of Eq. (2.7b). If we reverse the process, that is, if we start by taking the curl of both sides of Eq. (2.7d) and then use Eq. (2.7b) to eliminate **E**, we obtain a wave equation for **H**:

$$\nabla^2 \mathbf{H} - \gamma^2 \mathbf{H} = 0. \qquad (2.14)$$

Since the wave equations for **E** and **H** are of the same form, so are the forms of their solutions.

2-2 Plane-Wave Propagation in Lossless Media

The properties of an electromagnetic wave, such as its phase velocity u_p and wavelength λ, depend on the angular frequency ω and the medium's three constitutive parameters: ε', μ, and σ. If the medium is ***nonconducting*** ($\sigma = 0$), the wave does not suffer any attenuation as it travels and hence the medium is said to be ***lossless***. Because in a lossless medium $\varepsilon = \varepsilon'$, Eq. (2.12) becomes

$$\gamma^2 = -\omega^2 \mu \varepsilon' \varepsilon_0. \qquad (2.15)$$

For lossless media, it is customary to define the ***wavenumber*** k as

$$k = \omega \sqrt{\mu \varepsilon' \varepsilon_0}\,. \qquad (2.16)$$

In view of Eq. (2.15), $\gamma^2 = -k^2$ and Eq. (2.13) becomes

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0. \qquad (2.17)$$

2-2.1 Uniform Plane Waves

For an electric field phasor expressed in a Cartesian coordinate system as

$$\mathbf{E} = \hat{\mathbf{x}} E_x + \hat{\mathbf{y}} E_y + \hat{\mathbf{z}} E_z, \qquad (2.18)$$

substitution of Eq. (2.11) into Eq. (2.17) gives

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) (\hat{\mathbf{x}} E_x + \hat{\mathbf{y}} E_y + \hat{\mathbf{z}} E_z) + k^2 (\hat{\mathbf{x}} E_x + \hat{\mathbf{y}} E_y + \hat{\mathbf{z}} E_z) = 0. \qquad (2.19)$$

To satisfy Eq. (2.19), each vector component on the left-hand side of the equation must vanish. Hence,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) E_x = 0, \qquad (2.20)$$

and similar expressions apply to E_y and E_z.

▶ A ***uniform plane wave*** is characterized by electric and magnetic fields that have uniform properties at all points across an infinite plane. ◀

If this happens to be the x–y plane, then **E** and **H** do not vary with x and y. Hence, $\partial E_x/\partial x = 0$ and $\partial E_x/\partial y = 0$, and Eq. (2.20) reduces to

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0. \qquad (2.21)$$

Similar expressions apply to E_y, H_x, and H_y. The remaining components of **E** and **H** are zero; that is, $E_z = H_z = 0$.

▶ A plane wave has no electric- or magnetic-field components along its direction of propagation. ◀

For the phasor quantity E_x, the general solution of the ordinary differential equation given by Eq. (2.21) is

$$E_x(z) = E_x^+(z) + E_x^-(z) = E_{x0}^+ e^{-jkz} + E_{x0}^- e^{jkz}, \qquad (2.22)$$

where E_{x0}^+ and E_{x0}^- are constants to be determined from boundary conditions. The first term in Eq. (2.22), containing the negative exponential e^{-jkz}, represents a wave with amplitude E_{x0}^+ traveling in the $+z$ direction. Likewise, the second term (with e^{jkz}) represents a wave with amplitude E_{x0}^- traveling in the $-z$ direction. Assume for the time being that **E** only has a component along x (i.e., $E_y = 0$) and that E_x is associated with a wave traveling in the $+z$ direction only (i.e., $E_{x0}^- = 0$). Under these conditions,

$$\mathbf{E}(z) = \hat{\mathbf{x}} E_x^+(z) = \hat{\mathbf{x}} E_{x0}^+ e^{-jkz}. \tag{2.23}$$

To find the magnetic field $\mathbf{H}$ associated with this wave, we apply Eq. (2.7b) with $\mathbf{E}_y = \mathbf{E}_z = 0$. The process leads to

$$H_y(z) = H_{y0}^+ e^{-jkz}, \tag{2.24a}$$

with

$$H_{y0}^+ = \frac{k}{\omega\mu} E_{x0}^+, \tag{2.24b}$$

and the other components of $\mathbf{H}$ are zero (i.e., $H_x = H_z = 0$). The ***intrinsic impedance*** of a lossless medium is defined as

$$\eta = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon'\varepsilon_0}} = \sqrt{\frac{\mu}{\varepsilon'\varepsilon_0}} \quad (\Omega), \tag{2.25}$$

where we used the expression for k given by Eq. (2.16). In view of Eq. (2.25), the electric and magnetic fields of a $+z$-propagating plane wave with $\mathbf{E}$-field along $\hat{\mathbf{x}}$ are:

$$\mathbf{E}(z) = \hat{\mathbf{x}} E_x^+(z) = \hat{\mathbf{x}} E_{x0}^+ e^{-jkz}, \tag{2.26a}$$

$$\mathbf{H}(z) = \hat{\mathbf{y}} \frac{E_x^+(z)}{\eta} = \hat{\mathbf{y}} \frac{E_{x0}^+}{\eta} e^{-jkz}. \tag{2.26b}$$

▶ Because the electric and magnetic fields are perpendicular to each other, and both are perpendicular to the direction of wave travel, this wave is said to be ***transverse electromagnetic*** (TEM) (Fig. 2-2). ◀

In the general case, E_{x0}^+ is a complex quantity with magnitude $|E_{x0}^+|$ and phase angle ϕ^+. That is,

$$E_{x0}^+ = |E_{x0}^+| e^{j\phi^+}. \tag{2.27}$$

The instantaneous electric and magnetic fields therefore are

$$\mathbf{E}(z,t) = \mathfrak{Re}\left[\mathbf{E}(z)\, e^{j\omega t}\right] = \hat{\mathbf{x}} |E_{x0}^+| \cos(\omega t - kz + \phi^+) \quad \text{(V/m)}, \tag{2.28a}$$

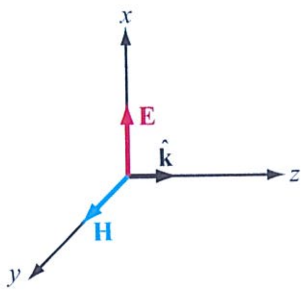


Figure 2-2: A transverse electromagnetic (TEM) wave propagating in the direction $\hat{\mathbf{k}} = \hat{\mathbf{z}}$. For all TEM waves, $\hat{\mathbf{k}}$ is parallel to $\mathbf{E} \times \mathbf{H}$.

and

$$\mathbf{H}(z,t) = \mathfrak{Re}\left[\mathbf{H}(z)\, e^{j\omega t}\right] = \hat{\mathbf{y}} \frac{|E_{x0}^+|}{\eta} \cos(\omega t - kz + \phi^+) \quad \text{(A/m)}. \tag{2.28b}$$

Because $\mathbf{E}(z,t)$ and $\mathbf{H}(z,t)$ exhibit the same functional dependence on z and t, they are said to be ***in phase***; when the amplitude of one of them reaches a maximum, the amplitude of the other does so too. The fact that $\mathbf{E}$ and $\mathbf{H}$ are in phase is characteristic of waves propagating in lossless media.

The ***phase velocity*** of the wave is

$$u_p = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\varepsilon'\varepsilon_0}} = \frac{1}{\sqrt{\mu\varepsilon'\varepsilon_0}} \quad \text{(m/s)}, \tag{2.29}$$

and its wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{u_p}{f} \quad \text{(m)}. \tag{2.30}$$

In vacuum, $\varepsilon' = 1$ and $\mu = \mu_0$, and the phase velocity u_p and the intrinsic impedance η given by Eq. (2.25) are

$$u_p = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 299{,}792{,}458 \approx 3 \times 10^8 \quad \text{(m/s)}, \tag{2.31}$$

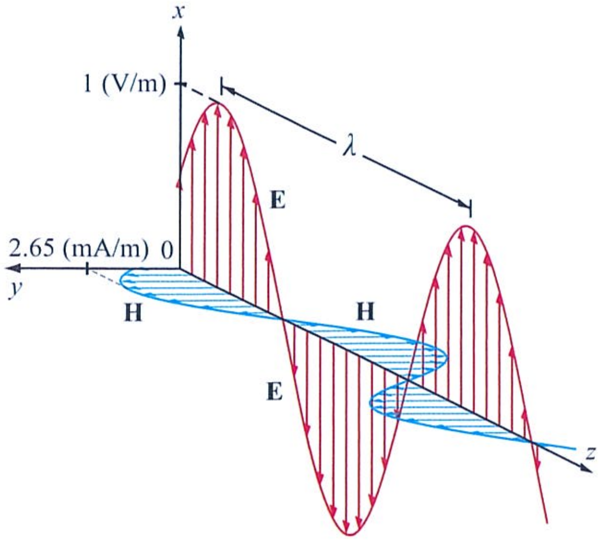


Figure 2-3: Spatial variations of **E** and **H** at $t = 0$ for the plane wave defined by Eq. (2.33).

$$\eta = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\ (\Omega) \approx 120\pi \quad (\Omega), \tag{2.32}$$

where c is the velocity of light and η_0 is called the ***intrinsic impedance of free space***.

Figure 2-3 displays the profiles of **E** and **H** for a 1 MHz EM wave traveling in air, with

$$\mathbf{E}(z,t) = \hat{\mathbf{x}} \cos\left(2\pi \times 10^6 t - \frac{2\pi z}{300} + \frac{\pi}{3}\right) \quad \text{(V/m)}, \tag{2.33a}$$

$$\begin{aligned} \mathbf{H}(z,t) &= \hat{\mathbf{y}}\,\frac{E(z,t)}{\eta_0} \\ &= \hat{\mathbf{y}}\, 2.65 \cos\left(2\pi \times 10^6 t - \frac{2\pi z}{300} + \frac{\pi}{3}\right) \quad \text{(mA/m)}. \end{aligned} \tag{2.33b}$$

2-2.2 General Relation between E and H

It can be shown that, for any uniform plane wave traveling in an arbitrary direction denoted by the unit vector $\hat{\mathbf{k}}$, the electric and magnetic field phasors **E** and **H** are related as

$$\mathbf{H} = \frac{1}{\eta}\,\hat{\mathbf{k}} \times \mathbf{E}, \tag{2.34a}$$

$$\mathbf{E} = -\eta\,\hat{\mathbf{k}} \times \mathbf{H}. \tag{2.34b}$$

▶ The following right-hand rule applies: when we rotate the four fingers of the right hand from the direction of **E** toward that of **H**, the thumb points in the direction of wave travel, $\hat{\mathbf{k}}$. ◀

The relations given by Eqs. (2.34a and b) are valid not only for lossless media, but for lossy media as well. As shown later in Section 2-4, the expression for η of a lossy medium is different from that given by Eq. (2.25). As long as the expression used for η is appropriate for the medium in which the wave is traveling, the relations given by Eqs. (2.34a and b) always hold.

Let us apply Eq. (2.34a) to the wave given by Eq. (2.26a). The direction of propagation $\hat{\mathbf{k}} = \hat{\mathbf{z}}$ and $\mathbf{E} = \hat{\mathbf{x}} E_x^+(z)$. Hence,

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E} = \frac{1}{\eta}(\hat{\mathbf{z}} \times \hat{\mathbf{x}}) E_x^+(z) = \hat{\mathbf{y}}\,\frac{E_x^+(z)}{\eta}, \tag{2.35}$$

which is the same as the result given by Eq. (2.26b).

In general, a uniform plane wave traveling in the $+z$ direction may have both x and y components, in which case **E** is given by

$$\mathbf{E} = \hat{\mathbf{x}} E_x^+(z) + \hat{\mathbf{y}} E_y^+(z), \tag{2.36a}$$

and the associated magnetic field is

$$\mathbf{H} = \hat{\mathbf{x}} H_x^+(z) + \hat{\mathbf{y}} H_y^+(z). \tag{2.36b}$$

Application of Eq. (2.34a) gives

$$\mathbf{H} = \frac{1}{\eta}\,\hat{\mathbf{z}} \times \mathbf{E} = -\hat{\mathbf{x}}\,\frac{E_y^+(z)}{\eta} + \hat{\mathbf{y}}\,\frac{E_x^+(z)}{\eta}. \tag{2.37}$$

By equating Eq. (2.36b) to Eq. (2.37), we have

$$H_x^+(z) = -\frac{E_y^+(z)}{\eta}, \qquad H_y^+(z) = \frac{E_x^+(z)}{\eta}. \tag{2.38}$$

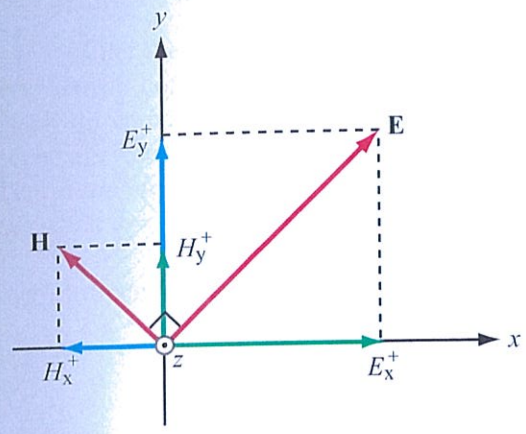


Figure 2-4: The wave $(\mathbf{E}, \mathbf{H})$ is equivalent to the sum of two waves, one with fields (E_x^+, H_y^+) and another with (E_y^+, H_x^+), with both traveling in the $+z$ direction.

These results are illustrated in Fig. 2-4. The wave may be considered to be the sum of two waves, one with electric and magnetic components (E_x^+, H_y^+), and another with components (E_y^+, H_x^+). In general, a TEM wave may have an electric field in any direction in the plane orthogonal to the direction of wave travel, and the associated magnetic field is also in the same plane and its direction is dictated by Eq. (2.34a).

2-3 Wave Polarization in a Lossless Medium

► The ***polarization*** of a uniform plane wave describes the locus traced by the tip of the **E** vector (in the plane orthogonal to the direction of propagation) at a given point in space as a function of time. ◄

In the most general case, the locus of the tip of **E** is an ellipse, and the wave is said to be ***elliptically polarized***. Under certain conditions, the ellipse may degenerate into a circle or a straight line, in which case the ***polarization state*** is called ***circular*** or ***linear***, respectively.

It was shown in Section 2-2 that the z components of the electric and magnetic fields of a z-propagating plane wave are both zero. Hence, in the most general case, the electric field phasor $\mathbf{E}(z)$ of a $+z$-propagating plane wave may consist of an x component, $\hat{\mathbf{x}}\, E_x(z)$, and a y component, $\hat{\mathbf{y}}\, E_y(z)$, or

$$\mathbf{E}(z) = \hat{\mathbf{x}} E_x(z) + \hat{\mathbf{y}} E_y(z), \tag{2.39}$$

with

$$E_x(z) = E_{x0} e^{-jkz}, \tag{2.40a}$$

$$E_y(z) = E_{y0} e^{-jkz}, \tag{2.40b}$$

where E_{x0} and E_{y0} are the amplitudes of $E_x(z)$ and $E_y(z)$, respectively. For the sake of simplicity, the plus sign superscript has been suppressed; the negative sign in e^{-jkz} is sufficient to remind us that the wave is traveling in the positive z direction.

The two amplitudes E_{x0} and E_{y0} are, in general, complex quantities, each characterized by a magnitude and a phase angle. The phase of a wave is defined relative to a reference state, such as $z = 0$ and $t = 0$ or any other combination of z and t. As will become clear from the discussion that follows, the polarization of the wave described by Eqs. (2.39) and (2.40) depends on the phase of E_{y0} relative to that of E_{x0}, but not on the absolute phases of E_{x0} and E_{y0}. Hence, for convenience, we assign E_{x0} a phase of zero and denote the phase of E_{y0}, relative to that of E_{x0}, as δ. Thus, δ is the *phase difference* between the y- and x components of **E**. Accordingly, we define E_{x0} and E_{y0} as

$$E_{x0} = a_x, \tag{2.41a}$$

$$E_{y0} = a_y e^{j\delta}, \tag{2.41b}$$

where $a_x = |E_{x0}| \geq 0$ and $a_y = |E_{y0}| \geq 0$ are the magnitudes of E_{x0} and E_{y0}, respectively. Thus, by definition, a_x and a_y may not assume negative values. Using Eqs. (2.41a) and (2.41b) in Eqs. (2.40a) and (2.40b), the total electric field phasor is

$$\mathbf{E}(z) = (\hat{\mathbf{x}} a_x + \hat{\mathbf{y}} a_y e^{j\delta}) e^{-jkz}, \tag{2.42}$$

and the corresponding instantaneous field is

$$\begin{aligned}\mathbf{E}(z,t) &= \mathfrak{Re}\left[\mathbf{E}(z)\,e^{j\omega t}\right]\\ &= \hat{\mathbf{x}}a_x\cos(\omega t - kz)\\ &\qquad + \hat{\mathbf{y}}a_y\cos(\omega t - kz + \delta). \qquad (2.43)\end{aligned}$$

When characterizing an electric field at a given point in space, two attributes of particular interest are its magnitude and direction. The magnitude of $\mathbf{E}(z,t)$ is

$$\begin{aligned}|\mathbf{E}(z,t)| &= [E_x^2(z,t) + E_y^2(z,t)]^{1/2}\\ &= [a_x^2\cos^2(\omega t - kz)\\ &\qquad + a_y^2\cos^2(\omega t - kz + \delta)]^{1/2}. \qquad (2.44)\end{aligned}$$

The electric field $\mathbf{E}(z,t)$ has components along the x- and y directions. At a specific position z, the direction of $\mathbf{E}(z,t)$ is characterized by its ***inclination angle*** $\tau(z,t)$, defined with respect to the x axis and given by

$$\tau(z,t) = \tan^{-1}\left[\frac{E_y(z,t)}{E_x(z,t)}\right]. \qquad (2.45)$$

In the general case, both the intensity of $\mathbf{E}(z,t)$ and its direction are functions of z and t. Next, we examine some special cases.

2-3.1 Linear Polarization

▶ A wave is said to be linearly polarized if for a fixed z, the tip of $\mathbf{E}(z,t)$ traces a straight line segment as a function of time. This happens when $E_x(z,t)$ and $E_y(z,t)$ are ***in phase*** (i.e., $\delta = 0$) or ***out of phase*** ($\delta = \pi$). ◀

Under these conditions Eq. (2.43) simplifies to

$$\mathbf{E}(z,t) = (\hat{\mathbf{x}}a_x + \hat{\mathbf{y}}a_y)\cos(\omega t - kz) \quad \textbf{(in phase)}, \qquad (2.46a)$$

$$\mathbf{E}(z,t) = (\hat{\mathbf{x}}a_x - \hat{\mathbf{y}}a_y)\cos(\omega t - kz) \quad \textbf{(out of phase)}. \qquad (2.46b)$$

Let us examine the out-of-phase case. The field's magnitude is

$$|\mathbf{E}(z,t)| = [a_x^2 + a_y^2]^{1/2}\,|\cos(\omega t - kz)|, \qquad (2.47a)$$

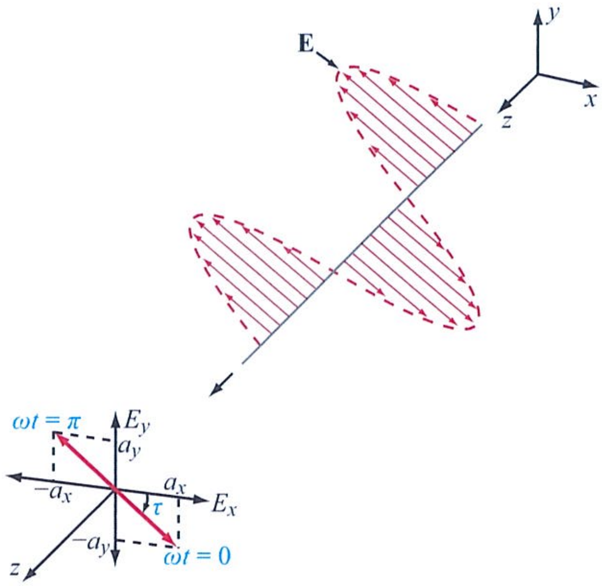


Figure 2-5: Linearly polarized wave traveling in the $+z$ direction.

and the inclination angle is

$$\begin{aligned}\tau(z,t) &= \tan^{-1}\left[\frac{-a_y\cos(\omega t - kz)}{a_x\cos(\omega t - kz)}\right]\\ &= \tan^{-1}\left(\frac{-a_y}{a_x}\right) \quad \textbf{(out of phase)}. \qquad (2.47b)\end{aligned}$$

We note that τ is independent of both z and t. Figure 2-5 displays the line segment traced by the tip of $\mathbf{E}$ at $z = 0$ over a half of a cycle. The trace would be the same at any other value of z as well. At $z = 0$ and $t = 0$, $|E(0,0)| = [a_x^2 + a_y^2]^{1/2}$. The length of the vector representing $\mathbf{E}(0,t)$ decreases to zero at $\omega t = \pi/2$. The vector then reverses direction and increases in magnitude to $[a_x^2 + a_y^2]^{1/2}$ in the second quadrant of the x–y plane at $\omega t = \pi$. Since τ is independent of both z and t, $\mathbf{E}(z,t)$ maintains a direction along the line making an angle τ with the x axis, while oscillating back and forth across the origin.

If $a_y = 0$, then $\tau = 0°$ or $180°$, and the wave is x polarized; conversely, if $a_x = 0$, then $\tau = 90°$ or $-90°$, and the wave is y polarized.

2-3.2 Circular Polarization

We now consider the special case when the magnitudes of the x- and y components of $\mathbf{E}(z)$ are equal, and the phase difference $\delta = \pm\pi/2$. For reasons that will become evident shortly, the wave polarization is called ***left-hand circular*** when $\delta = \pi/2$, and ***right-hand circular*** when $\delta = -\pi/2$.

Left-hand circular (LHC) polarization

For $a_x = a_y = a$ and $\delta = \pi/2$, Eqs. (2.42) and (2.43) become

$$\mathbf{E}(z) = (\hat{\mathbf{x}}a + \hat{\mathbf{y}}ae^{j\pi/2})e^{-jkz} = a(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-jkz}, \quad (2.48a)$$

$$\mathbf{E}(z,t) = \mathfrak{Re}\left[\mathbf{E}(z)\,e^{j\omega t}\right] = \hat{\mathbf{x}}a\cos(\omega t - kz) + \hat{\mathbf{y}}a\cos(\omega t - kz + \pi/2) = \hat{\mathbf{x}}a\cos(\omega t - kz) - \hat{\mathbf{y}}a\sin(\omega t - kz). \quad (2.48b)$$

The corresponding field magnitude and inclination angle are

$$|\mathbf{E}(z,t)| = \left[E_x^2(z,t) + E_y^2(z,t)\right]^{1/2} = [a^2\cos^2(\omega t - kz) + a^2\sin^2(\omega t - kz)]^{1/2} = a \quad (2.49a)$$

and

$$\tau(z,t) = \tan^{-1}\left[\frac{E_y(z,t)}{E_x(z,t)}\right] = \tan^{-1}\left[\frac{-a\sin(\omega t - kz)}{a\cos(\omega t - kz)}\right] = -(\omega t - kz). \quad (2.49b)$$

We observe that the magnitude of $\mathbf{E}$ is independent of both z and t, whereas $\tau(z,t)$ depends on both variables. These functional dependencies are the converse of those for the linear polarization case.

At $z = 0$, Eq. (2.49b) gives $\tau = -\omega t$; the negative sign implies that the inclination angle decreases as time increases. As illustrated in Fig. 2-6(a), the tip of $\mathbf{E}(t)$ traces a circle in the x–y plane and rotates in a clockwise direction as a function of time (when viewing the wave approaching). Such a wave is called ***left-hand circularly polarized***, because when the thumb of the left hand points along the direction of propagation (the z direction in this case), the other four fingers curl in the direction of rotation of $\mathbf{E}$.

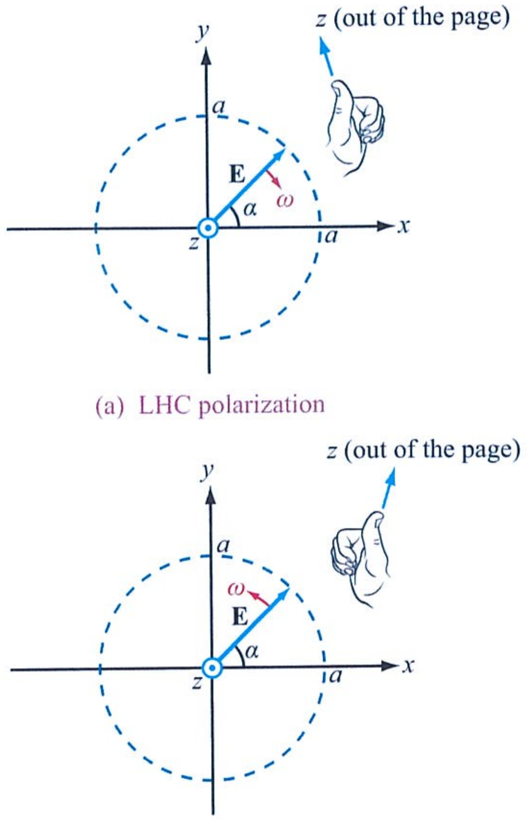


Figure 2-6: Circularly polarized plane waves propagating in the $+z$ direction (out of the page).

Right-hand circular (RHC) polarization

For $a_x = a_y = a$ and $\delta = -\pi/2$, we have

$$|\mathbf{E}(z,t)| = a, \qquad \tau = (\omega t - kz). \quad (2.50)$$

The trace of $\mathbf{E}(0,t)$ as a function of t is shown in Fig. 2-6(b). For RHC polarization, the fingers of the

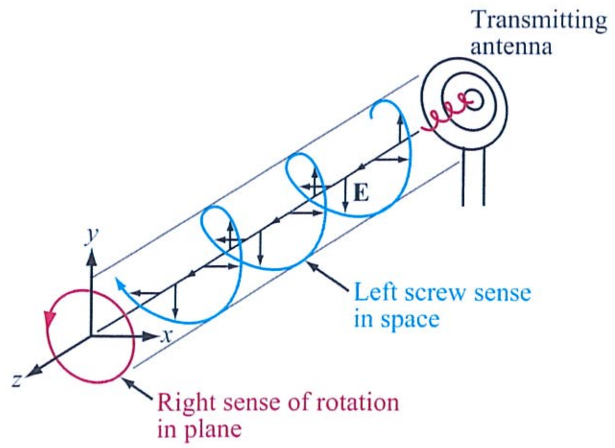


Figure 2-7: Right-hand circularly polarized wave radiated by a helical antenna.

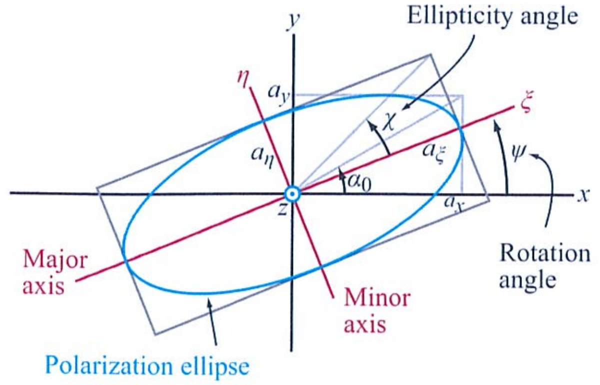


Figure 2-8: Polarization ellipse in the x–y plane, with the wave traveling in the z direction (out of the page).

right hand curl in the direction of rotation of **E** when the thumb is along the propagation direction. Figure 2-7 depicts a right-hand circularly polarized wave radiated by a helical antenna.

▶ Polarization handedness is defined by the rotation of **E** as a function of time in a fixed plane orthogonal to the direction of propagation, which is opposite of the direction of rotation of **E** as a function of distance at a fixed point in time. ◀

2-3.3 Elliptical Polarization

Plane waves that are not linearly or circularly polarized are elliptically polarized. That is, the tip of $\mathbf{E}(z,t)$ traces an ellipse in the plane perpendicular to the direction of propagation. The shape of the ellipse and the field's handedness (left-hand or right-hand) are determined by the values of the ratio (a_y/a_x) and the phase difference δ.

The polarization ellipse shown in Fig. 2-8 has its major axis with length a_ξ along the ξ direction and its minor axis with length a_η along the η direction. The ***rotation angle*** ψ is defined as the angle between the major axis of the ellipse and a reference direction, chosen here to be the x axis, with ψ being bounded within the range $-\pi/2 \le \psi \le \pi/2$. The shape of the ellipse and its handedness are characterized by the ***ellipticity angle*** χ, defined as

$$\tan\chi = \pm\frac{a_\eta}{a_\xi} = \pm\frac{1}{R}, \tag{2.51}$$

with the plus sign corresponding to left-handed rotation and the minus sign corresponding to right-handed rotation. The limits for χ are $-\pi/4 \le \chi \le \pi/4$. The quantity $R = a_\xi/a_\eta$ is called the ***axial ratio*** of the polarization ellipse, and it varies between 1 for circular polarization and ∞ for linear polarization. The polarization angles ψ and χ are related to the wave parameters a_x, a_y, and δ by (Born and Wolf, 1965)

$$\tan 2\psi = (\tan 2\alpha_0)\cos\delta \quad (-\pi/2 \le \psi \le \pi/2), \tag{2.52a}$$

$$\sin 2\chi = (\sin 2\alpha_0)\sin\delta \quad (-\pi/4 \le \chi \le \pi/4), \tag{2.52b}$$

where α_0 is an ***auxiliary angle*** defined by

$$\tan\alpha_0 = \frac{a_y}{a_x} \qquad \left(0 \le \alpha_0 \le \frac{\pi}{2}\right). \tag{2.53}$$

Sketches of the polarization ellipse are shown in Fig. 2-9 for various combinations of the angles (ψ, χ). The ellipse reduces to a circle for $\chi = \pm 45°$ and to a line for

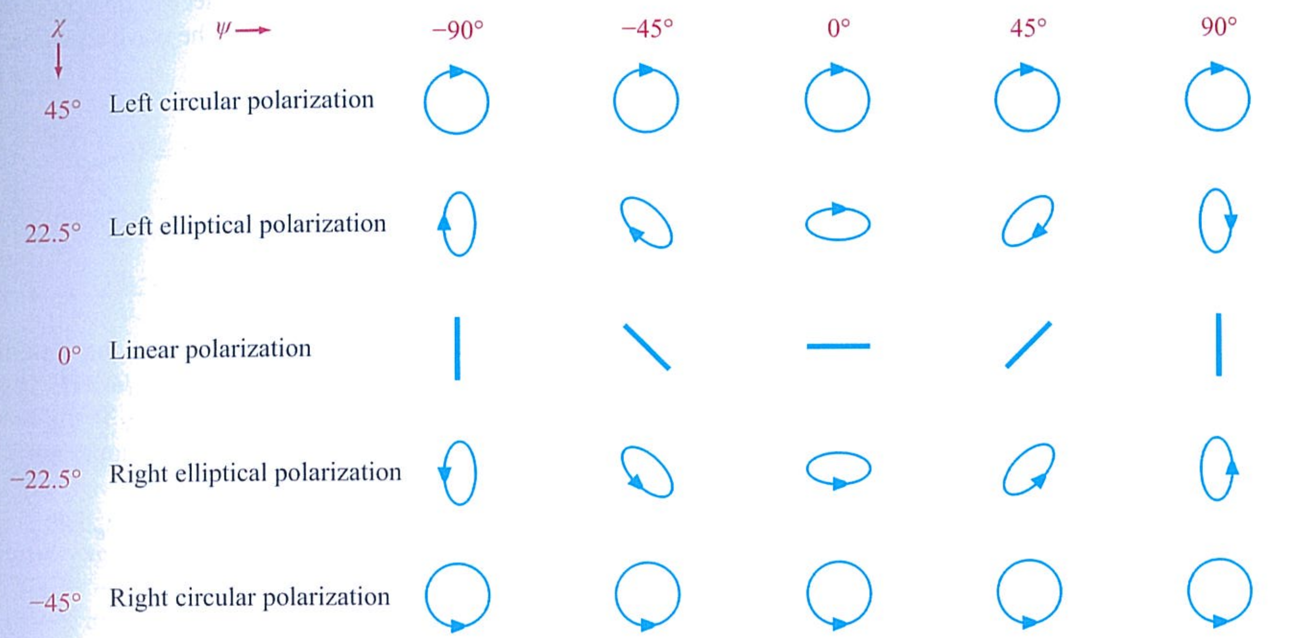


Figure 2-9: Polarization states for various combinations of the polarization angles (ψ, χ) for a wave traveling out of the page.†

$\chi = 0$. *Positive values of χ, corresponding to $\sin\delta > 0$, are associated with left-handed rotation, and negative values of χ, corresponding to $\sin\delta < 0$, are associated with right-handed rotation.*

Since the magnitudes a_x and a_y are, by definition, nonnegative numbers, the ratio a_y/a_x may vary between zero for an x-polarized linear polarization and ∞ for a y-polarized linear polarization. Consequently, the angle α_0 is limited to the range $0 \leq \alpha_0 \leq 90°$. Application of Eq. (2.52a) leads to two possible solutions for the value of ψ, both of which fall within the defined range from $-\pi/2$ to $\pi/2$. The correct choice is governed by the following rule:

$$\psi > 0 \text{ if } \cos\delta > 0,$$
$$\psi < 0 \text{ if } \cos\delta < 0.$$

▶ In summary, the sign of the rotation angle ψ is the same as the sign of $\cos\delta$ and the sign of the ellipticity angle χ is the same as the sign of $\sin\delta$. ◀

2-4 Plane-Wave Propagation in Lossy Media

To examine wave propagation in a lossy (conducting) medium, we return to the wave equation given by Eq. (2.13),

$$\nabla^2 \mathbf{E} - \gamma^2 \mathbf{E} = 0,$$

with

$$\gamma^2 = -\omega^2 \mu \varepsilon \varepsilon_0 = -\omega^2 \mu \varepsilon_0 (\varepsilon' - j\varepsilon''), \tag{2.54}$$

where $\varepsilon'' = \sigma/\omega\varepsilon_0$. Since γ is complex, we express it as

$$\gamma = \alpha + j\beta, \tag{2.55}$$

† Computer Code 2.1 (see page vii).

where α is the medium's ***attenuation constant*** and β its ***phase constant***. The combination of Eqs. (2.54) and (2.55) leads to

$$\alpha = -\omega\sqrt{\mu\varepsilon_0}\ \mathfrak{Im}\left\{\sqrt{\varepsilon}\right\}, \tag{2.56a}$$

$$\beta = \omega\sqrt{\mu\varepsilon_0}\ \mathfrak{Re}\left\{\sqrt{\varepsilon}\right\}. \tag{2.56b}$$

Alternatively, by replacing γ with $(\alpha + j\beta)$ in Eq. (2.54), we obtain

$$\begin{aligned}(\alpha + j\beta)^2 &= (\alpha^2 - \beta^2) + j2\alpha\beta \\ &= -\omega^2\mu\varepsilon'\varepsilon_0 + j\omega^2\mu\varepsilon''\varepsilon_0.\end{aligned} \tag{2.57}$$

The rules of complex algebra require the real and imaginary parts on one side of an equation to equal the real and imaginary parts on the other side, which leads to

$$\begin{aligned}\alpha &= \omega\left\{\frac{\mu_0\varepsilon'\varepsilon_0}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right\}^{1/2} \\ &= \frac{2\pi}{\lambda_0}\left\{\frac{\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right\}^{1/2} \quad \text{(Np/m)},\end{aligned} \tag{2.58a}$$

$$\begin{aligned}\beta &= \omega\left\{\frac{\mu_0\varepsilon'\varepsilon_0}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right\}^{1/2} \\ &= \frac{2\pi}{\lambda_0}\left\{\frac{\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right\}^{1/2} \quad \text{(Np/m)},^{\dagger}\end{aligned} \tag{2.58b}$$

where $\lambda_0 = c/f$ is the wavelength in free space.

> ▶ We set $\mu = \mu_0$ because the natural materials encountered in remote sensing are nonmagnetic. This includes water, ice, soil, and vegetation, among many others. ◀

[†]**Computer Code 2.2.**

For a uniform plane wave with electric field $\mathbf{E} = \hat{\mathbf{x}}\, E_x(z)$ traveling along the $+z$ direction, the wave equation given by Eq. (2.13) reduces to

$$\frac{d^2 E_x(z)}{dz^2} - \gamma^2 E_x(z) = 0, \tag{2.59}$$

and its solution gives

$$\mathbf{E}(z) = \hat{\mathbf{x}} E_{x0} e^{-\gamma z} = \hat{\mathbf{x}} E_{x0} e^{-\alpha z} e^{-j\beta z}. \tag{2.60}$$

The associated magnetic field $\mathbf{H}$ can be determined by applying Eq. (2.3b): $\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$, or using Eq. (2.34a): $\mathbf{H} = (\hat{\mathbf{k}} \times \mathbf{E})/\eta_c$, where η_c is the ***intrinsic impedance of the lossy medium***. Both approaches give

$$\mathbf{H}(z) = \hat{\mathbf{y}}\, H_y(z) = \hat{\mathbf{y}}\, \frac{E_x(z)}{\eta_c} = \hat{\mathbf{y}}\, \frac{E_{x0}}{\eta_c} e^{-\alpha z} e^{-j\beta z}, \tag{2.61}$$

where

$$\begin{aligned}\eta_c = \sqrt{\frac{\mu_0}{\varepsilon\varepsilon_0}} &= \sqrt{\frac{\mu_0}{\varepsilon'\varepsilon_0}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} \\ &= \frac{\eta_0}{\sqrt{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} \quad (\Omega),^{\dagger}\end{aligned} \tag{2.62}$$

where η_0 is the intrinsic impedance of free space. We noted earlier that in a lossless medium, $\mathbf{E}(z,t)$ is in phase with $\mathbf{H}(z,t)$; however, this property no longer holds true in a lossy medium because η_c is complex.

From Eq. (2.60), the magnitude of $E_x(z)$ is given by

$$|E_x(z)| = |E_{x0} e^{-\alpha z} e^{-j\beta z}| = |E_{x0}| e^{-\alpha z}, \tag{2.63}$$

which decreases exponentially with z at a rate dictated by the attenuation constant α. Since $H_y = E_x/\eta_c$, the magnitude of H_y also decreases as $e^{-\alpha z}$. As the field attenuates, part of the energy carried by the electromagnetic wave is converted into heat due to conduction in the medium. As the wave travels through a distance $z = \delta_s$ with

$$\delta_s = \frac{1}{\alpha} \quad \text{(m)}, \tag{2.64}$$

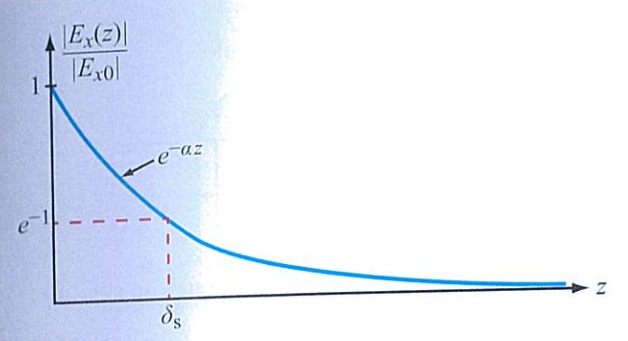


Figure 2-10: Attenuation of the magnitude of $E_x(z)$ with distance z. The skin depth δ_s is the value of z at which $|E_x(z)|/|E_{x0}| = e^{-1}$, or $z = \delta_s = 1/\alpha$.

the wave magnitude decreases by a factor of $e^{-1} \approx 0.37$ (Fig. 2-10). At depth $z = 3\delta_s$, the field magnitude is less than 5% of its initial value, and at $z = 5\delta_s$, it is less than 1%.

> ▶ This distance δ_s, called the ***skin depth*** of the medium, characterizes how deep an electromagnetic wave can penetrate into a conducting medium. ◀

In a perfect dielectric, $\sigma = 0$ and $\varepsilon'' = 0$; use of Eq. (2.58a) yields $\alpha = 0$ and therefore $\delta_s = \infty$. Thus, in free space, a plane wave can propagate indefinitely with no loss in magnitude. On the other extreme, in a perfect conductor, $\sigma = \infty$ and use of Eq. (2.58a) leads to $\alpha = \infty$ and hence $\delta_s = 0$. Thus, the electric field is confined to the surface of a perfect conductor.

The expressions given by Eqs. (2.58a), (2.58b), and (2.62) for α, β, and η_c are valid for any linear, isotropic, and homogeneous medium. For a perfect dielectric ($\sigma = 0$), these expressions reduce to those for the lossless case (Section 2-2), wherein $\alpha = 0$, $\beta = k = \omega\sqrt{\mu_0\varepsilon'\varepsilon_0}$, and $\eta_c = \eta$. For a lossy medium, the ratio $\varepsilon''/\varepsilon' = \sigma/\omega\varepsilon'\varepsilon_0$, which appears in all these expressions, plays an important role in classifying how lossy the medium is. When $\varepsilon''/\varepsilon' \ll 1$, the medium is considered a ***low-loss dielectric***, and when $\varepsilon''/\varepsilon' \gg 1$, it is considered a ***good conductor***. In practice, the medium may be regarded as a low-loss dielectric if $\varepsilon''/\varepsilon' < 10^{-2}$, as a good conductor if $\varepsilon''/\varepsilon' > 10^2$, and as a ***quasi conductor*** if $10^{-2} \le \varepsilon''/\varepsilon' \le 10^2$. For low-loss dielectrics and good conductors, the expressions given by Eq. (2.58) can be significantly simplified, as shown next.

2-4.1 Low-Loss Dielectric

From Eq. (2.54), the general expression for γ in a nonmagnetic medium is

$$\gamma = j\omega\sqrt{\mu_0\varepsilon'\varepsilon_0}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{1/2}. \qquad (2.65)$$

For $|x| \ll 1$, the function $(1-x)^{1/2}$ can be approximated by the first two terms of its binomial series; that is, $(1-x)^{1/2} \approx 1 - x/2$. By applying this approximation to Eq. (2.65) for a low-loss dielectric with $x = j\varepsilon''/\varepsilon'$ and $\varepsilon''/\varepsilon' \ll 1$, we obtain

$$\gamma \approx j\omega\sqrt{\mu_0\varepsilon'\varepsilon_0}\left(1 - j\frac{\varepsilon''}{2\varepsilon'}\right)$$
$$\approx j\frac{2\pi}{\lambda_0}\sqrt{\varepsilon'}\left(1 - j\frac{\varepsilon''}{2\varepsilon'}\right). \qquad (2.66)$$

The real and imaginary parts of Eq. (2.66) are

$$\alpha \approx \frac{\pi\varepsilon''}{\lambda_0\sqrt{\varepsilon'}} \quad \text{(Np/m)}, \qquad (2.67a)$$

$$\beta \approx \frac{2\pi}{\lambda_0}\sqrt{\varepsilon'} \quad \text{(rad/m)}. \qquad (2.67b)$$

We note that the expression for β is the same as that for the wavenumber k of a lossless medium. Applying the binomial approximation $(1-x)^{-1/2} \approx (1 + x/2)$ to Eq. (2.62) leads to

$$\eta_c \approx \frac{\eta_0}{\sqrt{\varepsilon'}}\left(1 + j\frac{\varepsilon''}{2\varepsilon'}\right). \qquad (2.68a)$$

In practice, because $\varepsilon''/\varepsilon' < 10^{-2}$ for low-loss materials, the second term in Eq. (2.68a) often is ignored. Thus,

$$\eta_c \approx \frac{\eta_0}{\sqrt{\varepsilon'}}, \qquad (2.68b)$$

which is the same as Eq. (2.25) for a lossless nonmagnetic material.

2-4.2 Good Conductor

When $\varepsilon''/\varepsilon' > 100$, Eqs. (2.58a), (2.58b), and (2.62) can be approximated as

$$\alpha \approx \frac{\pi\sqrt{2\varepsilon''}}{\lambda_0} \quad \text{(Np/m)}, \tag{2.69a}$$

$$\beta = \alpha \approx \frac{\pi\sqrt{2\varepsilon''}}{\lambda_0} \quad \text{(rad/m)}, \tag{2.69b}$$

$$\eta_c \approx \sqrt{\frac{j\mu_0}{\varepsilon''\varepsilon_0}} = \frac{(1+j)\eta_0}{\sqrt{2\varepsilon''}} \quad (\Omega). \tag{2.69c}$$

In Eq. (2.69c), we used the relation $\sqrt{j} = (1+j)/\sqrt{2}$. For a perfect conductor with $\sigma = \infty$, these expressions yield $\alpha = \beta = \infty$, and $\eta_c = 0$.

Expressions for the propagation parameters in various types of media are summarized in Table 2-1.

2-5 Electromagnetic Power Density

This section deals with the flow of power carried by an electromagnetic wave. For any wave with time-domain electric field $\mathbf{E}(t)$ and associated magnetic field $\mathbf{H}(t)$, the ***Poynting vector*** $\mathbf{S}(t)$ is defined as

$$\mathbf{S}(t) = \mathbf{E}(t) \times \mathbf{H}(t) \quad \text{(W/m}^2\text{)}. \tag{2.70}$$

The unit of $\mathbf{S}(t)$ is (V/m) × (A/m) = (W/m^2), and the direction of $\mathbf{S}(t)$ is along the wave's direction of propagation. Thus, $\mathbf{S}(t)$ represents the power per unit area (or power density) carried by the wave.

In practice, the quantity of greater interest is the ***average power density*** of the wave, $\mathbf{S}$, which is the time-average value of $\mathbf{S}(t)$:

$$\mathbf{S} = \tfrac{1}{2}\mathfrak{Re}\left[\mathbf{E} \times \mathbf{H}^*\right] \quad \text{(W/m}^2\text{)}. \tag{2.71}$$

If the wave is incident upon an aperture of area A with outward surface unit vector $\hat{\mathbf{n}}$ as shown in Fig. 2-11, then

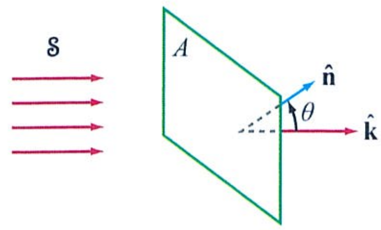


Figure 2-11: EM power flow through an aperture.

the total power that flows through or is intercepted by the aperture is

$$P = \int_A \mathbf{S} \cdot \hat{\mathbf{n}}\, dA \quad \text{(W)}. \tag{2.72}$$

For a uniform plane wave propagating in a direction $\hat{\mathbf{k}}$ that makes an angle θ with $\hat{\mathbf{n}}$, $P = SA\cos\theta$, where $S = |\mathbf{S}|$.

2-5.1 Plane Wave in a Lossless Medium

Recall that the general expression for the electric field of a uniform plane wave with arbitrary polarization traveling in the $+z$ direction is

$$\begin{aligned}\mathbf{E}(z) &= \hat{\mathbf{x}}E_x(z) + \hat{\mathbf{y}}E_y(z) \\ &= (\hat{\mathbf{x}}E_{x0} + \hat{\mathbf{y}}E_{y0})e^{-jkz},\end{aligned} \tag{2.73}$$

where, in the general case, E_{x0} and E_{y0} are complex quantities. The magnitude of $\mathbf{E}$ is

$$|\mathbf{E}| = (\mathbf{E}\cdot\mathbf{E}^*)^{1/2} = [|E_{x0}|^2 + |E_{y0}|^2]^{1/2}. \tag{2.74}$$

The phasor magnetic field associated with $\mathbf{E}$ is obtained by applying Eq. (2.34a):

$$\begin{aligned}\mathbf{H}(z) &= (\hat{\mathbf{x}}H_x + \hat{\mathbf{y}}H_y)e^{-jkz} \\ &= \frac{1}{\eta}\,\hat{\mathbf{z}} \times \mathbf{E} = \frac{1}{\eta}\,(-\hat{\mathbf{x}}E_{y0} + \hat{\mathbf{y}}E_{x0})e^{-jkz}.\end{aligned} \tag{2.75}$$

The wave can be considered as the sum of two waves, one comprising fields (E_x, H_y) and another comprising fields (E_y, H_x). Use of Eqs. (2.73) and (2.75) in

Table 2-1: **Expressions for α, β, η_c, u_p, and λ for various types of nonmagnetic media.**[†]

	Any Medium	Lossless Medium ($\sigma = 0$)	Low-Loss Medium ($\varepsilon''/\varepsilon' \ll 1$)	Good Conductor ($\varepsilon''/\varepsilon' \gg 1$)	Units
$\alpha =$	$\omega\left[\frac{\mu_0\varepsilon'\varepsilon_0}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right]^{1/2}$	0	$\frac{\pi\varepsilon''}{\lambda_0\sqrt{\varepsilon'}}$	$\frac{\pi\sqrt{2\varepsilon''}}{\lambda_0}$	(Np/m)
$\beta =$	$\omega\left[\frac{\mu_0\varepsilon'\varepsilon_0}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right]^{1/2}$	$\frac{2\pi\sqrt{\varepsilon'}}{\lambda_0}$	$\frac{2\pi\sqrt{\varepsilon'}}{\lambda_0}$	$\frac{\pi\sqrt{2\varepsilon''}}{\lambda_0}$	(rad/m)
$\eta_c =$	$\sqrt{\frac{\mu_0}{\varepsilon'\varepsilon_0}}\left(1-j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\frac{\eta_0}{\sqrt{\varepsilon'}}$	$\frac{\eta_0}{\sqrt{\varepsilon'}}$	$\frac{(1+j)\eta_0}{\sqrt{2\varepsilon''}}$	(Ω)
$u_p =$	ω/β	$c/\sqrt{\varepsilon'}$	$c/\sqrt{\varepsilon'}$	$c\sqrt{2/\varepsilon''}$	(m/s)
$\lambda =$	$2\pi/\beta = u_p/f$	u_p/f	u_p/f	u_p/f	(m)

Notes: In practice, a material is considered a low-loss medium if $\varepsilon''/\varepsilon' < 0.01$ and a good conducting medium if $\varepsilon''/\varepsilon' > 100$; $c = 3 \times 10^8$ m/s; $\eta_0 = 377\ \Omega$.

Eq. (2.71) leads to

$$\mathcal{S} = \hat{\mathbf{z}}\,\frac{1}{2\eta}(|E_{x0}|^2 + |E_{y0}|^2) = \hat{\mathbf{z}}\,\frac{|\mathbf{E}|^2}{2\eta} \quad (\text{W/m}^2), \tag{2.76}$$

which states that power flows in the z direction with average power density equal to the sum of the average power densities of the (E_x, H_y) and (E_y, H_x) waves. Note that, because $\mathcal{S}$ depends only on η and $|\mathbf{E}|$, waves characterized by different polarizations carry the same amount of average power as long as their electric fields have the same magnitudes.

2-5.2 Plane Wave in a Lossy Medium

The expressions given by Eqs. (2.60) and (2.61) characterize the electric and magnetic fields of an x-polarized plane wave propagating along the z direction in a lossy medium with propagation constant $\gamma = \alpha + j\beta$. By extending these expressions to the more general case of a wave with components along both x and y, we have

$$\mathbf{E}(z) = \hat{\mathbf{x}}E_x(z) + \hat{\mathbf{y}}E_y(z) = (\hat{\mathbf{x}}E_{x0} + \hat{\mathbf{y}}E_{y0})e^{-\alpha z}e^{-j\beta z}, \tag{2.77a}$$

$$\mathbf{H}(z) = \frac{1}{\eta_c}(-\hat{\mathbf{x}}E_{y0} + \hat{\mathbf{y}}E_{x0})e^{-\alpha z}e^{-j\beta z}, \tag{2.77b}$$

where η_c is the intrinsic impedance of the lossy medium. Application of Eq. (2.71) gives

$$\mathcal{S}(z) = \frac{1}{2}\mathfrak{Re}\,[\mathbf{E}\times\mathbf{H}^*] = \frac{\hat{\mathbf{z}}(|E_{x0}|^2 + |E_{y0}|^2)}{2}\,e^{-2\alpha z}\,\mathfrak{Re}\left(\frac{1}{\eta_c^*}\right). \tag{2.78}$$

By expressing η_c in polar form as

$$\eta_c = |\eta_c|e^{j\theta_\eta}, \tag{2.79}$$

[†]Computer Code 2.1.

Eq. (2.78) can be rewritten as

$$\begin{aligned}\mathcal{S}(z) &= \hat{\mathbf{z}}\,\frac{|E(0)|^2}{2|\eta_c|}\,e^{-2\alpha z}\cos\theta_\eta \\ &= \hat{\mathbf{z}}\mathcal{S}_0 e^{-2\alpha z} \qquad (\text{W/m}^2),\end{aligned} \tag{2.80}$$

where $|E(0)|^2 = [|E_{x0}|^2 + |E_{y0}|^2]^{1/2}$ is the magnitude of $\mathbf{E}(z)$ at $z = 0$.

▶ Whereas the fields $\mathbf{E}(z)$ and $\mathbf{H}(z)$ decay with z as $e^{-\alpha z}$, the power density $\mathbf{S}$ decreases as $e^{-2\alpha z}$. ◀

When a wave propagates through a distance $z = \delta_s = 1/\alpha$, the magnitudes of its electric and magnetic fields decrease to $e^{-1} \approx 37\%$ of their initial values, and its average power density decreases to $e^{-2} \approx 14\%$ of its initial value.

2-5.3 Decibel Scale for Power Ratios

The unit for power P is watts (W). In many engineering problems, the quantity of interest is the ratio of two power levels, P_1 and P_2, such as the received and transmitted powers for a radar system, and often the ratio P_1/P_2 may vary over several orders of magnitude. The decibel (dB) scale is logarithmic, thereby providing a convenient representation of the power ratio, particularly when numerical values of P_1/P_2 are plotted against some variable of interest. If

$$G = \frac{P_1}{P_2}, \tag{2.81}$$

then

$$G\text{ [dB]} = 10\log G = 10\log\left(\frac{P_1}{P_2}\right) \qquad (\text{dB}). \tag{2.82}$$

The ***attenuation rate***, representing the rate of decrease of the magnitude of $\mathcal{S}(z)$ as a function of propagation distance, is defined as

$$\begin{aligned}A &= 10\log\left[\frac{\mathcal{S}(z)}{\mathcal{S}(0)}\right] \\ &= 10\log(e^{-2\alpha z}) \\ &= -20\alpha z\log e \\ &= -8.68\alpha z = -\alpha\,[\text{dB/m}]\,z \qquad (\text{dB}),\end{aligned} \tag{2.83}$$

where

$$\alpha\,[\text{dB/m}] = 8.68\alpha\,[\text{Np/m}]. \tag{2.84}$$

We also note that, since $\mathcal{S}(z)$ is directly proportional to $|\mathbf{E}(z)|^2$,

$$A = 10\log\left[\frac{|E(z)|^2}{|E(0)|^2}\right] = 20\log\left[\frac{|E(z)|}{|E(0)|}\right] \qquad (\text{dB}). \tag{2.85}$$

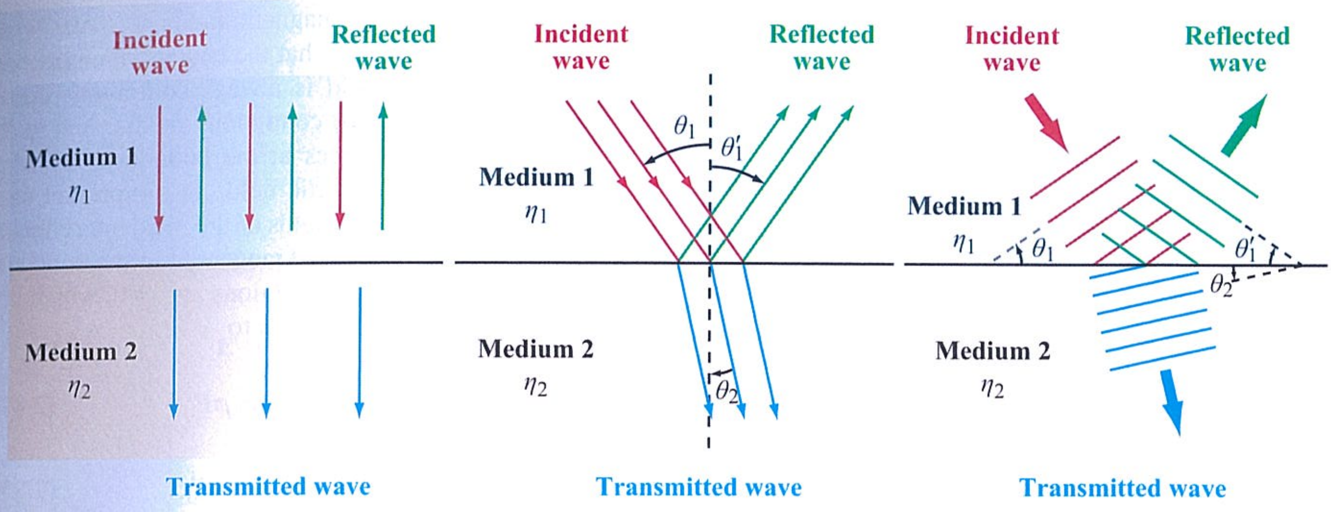
Table 2-2 compares selected values of G with the corresponding values of G [dB].

Table 2-2: Power ratios in natural numbers and in decibels.

G	G [dB]
10^x	$10x$ dB
4	6 dB
2	3 dB
1	0 dB
0.5	−3 dB
0.25	−6 dB
0.1	−10 dB
10^{-3}	−30 dB

2-6 Wave Reflection and Transmission at Normal Incidence

For convenience, we divide our treatment of wave reflection by, and transmission through, planar boundaries into three parts: in this section we confine our discussion to the normal-incidence case depicted in Fig. 2-12(a), in



(a) Normal incidence

(b) Ray representation of oblique incidence

(c) Wavefront representation of oblique incidence

Figure 2-12: Ray representation of wave reflection and transmission at (a) normal incidence and (b) oblique incidence, and (c) wavefront representation of oblique incidence.

Section 2-7 to 2-8 we examine the more general oblique-incidence case depicted in Fig. 2-12(b), and we do so for two different wave polarizations, and in Sections 2-9 and 2-10 we treat special cases of particular interest in remote sensing.

Before proceeding, however, we should explain the notion of rays and wavefronts, and the relationship between them, as both are used throughout this chapter to represent electromagnetic waves. A ***ray*** is a line representing the direction of flow of electromagnetic energy carried by a wave, and therefore it is parallel to the propagation unit vector $\hat{\mathbf{k}}$. A ***wavefront*** is a surface across which the phase of a wave is constant; it is perpendicular to vector $\hat{\mathbf{k}}$. Hence, rays are perpendicular to wavefronts. The ray representation of wave incidence, reflection, and transmission shown in Fig. 2-12(b) is equivalent to the wavefront representation depicted in Fig. 2-12(c). The two representations are complimentary; the ray representation is easier to use in graphical illustrations, whereas the wavefront representation provides greater physical insight into what happens to a wave when it encounters a discontinuous boundary. Both representations are used in forthcoming discussions.

2-6.1 Boundary between Lossless Media

A planar boundary located at $z = 0$ (Fig. 2-13) separates two lossless, homogeneous, dielectric media. Medium 1 has relative permittivity ε_1' and permeability μ_0 and fills the half-space $z \geq 0$. Medium 2 has relative permittivity ε_2' and permeability μ_0 and fills the half-space $z \leq 0$. A y-polarized plane wave, with electric and magnetic fields $(\mathbf{E}^{\mathrm{i}}, \mathbf{H}^{\mathrm{i}})$ propagates in medium 1 along direction $\hat{\mathbf{k}}_{\mathrm{i}} = -\hat{\mathbf{z}}$ toward medium 2. Reflection and transmission at the boundary at $z = 0$ result in a reflected wave, with electric and magnetic fields $(\mathbf{E}^{\mathrm{r}}, \mathbf{H}^{\mathrm{r}})$, traveling along direction $\hat{\mathbf{k}}_{\mathrm{r}} = \hat{\mathbf{z}}$ in medium 1, and a transmitted wave, with electric and magnetic fields $(\mathbf{E}^{\mathrm{t}}, \mathbf{H}^{\mathrm{t}})$, traveling along direction $\hat{\mathbf{k}}_{\mathrm{t}} = -\hat{\mathbf{z}}$ in medium 2. The expressions for the electric and magnetic fields of the three linearly polarized waves (Fig. 2-13) are given

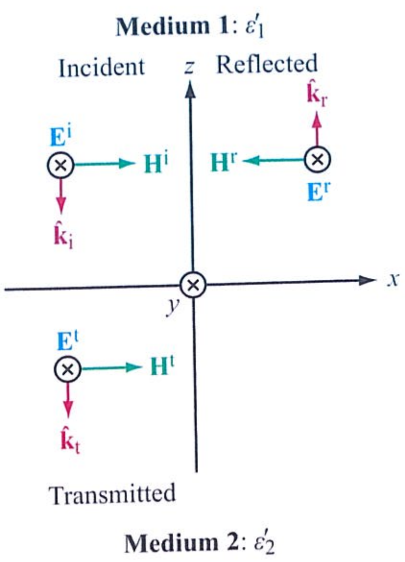


Figure 2-13: Two dielectric media separated by the x–y plane.

in the left-hand section of Table 2-3. The ***total fields*** in media 1 and 2 are

$$\mathbf{E}_1 = \mathbf{E}^i + \mathbf{E}^r, \qquad (2.86a)$$

$$\mathbf{H}_1 = \mathbf{H}^i + \mathbf{H}^r, \qquad (2.86b)$$

$$\mathbf{E}_2 = \mathbf{E}^t, \qquad (2.86c)$$

$$\mathbf{H}_2 = \mathbf{H}^t. \qquad (2.86d)$$

Note that superscripts i, r, and t are used to denote incident, reflected, and transmitted, and subscripts 1 and 2 denote the total fields in media 1 and 2. The amplitudes of the incident, reflected, and transmitted electric fields at $z = 0$ (the boundary between the two media) are E_0^i, E_0^r, and E_0^t, respectively. The wavenumber and intrinsic impedance of medium 1 are $k_1 = 2\pi\sqrt{\varepsilon'_1}/\lambda_0$ and $\eta_1 = \eta_0/\sqrt{\varepsilon'_1}$, and those for medium 2 are $k_2 = 2\pi\sqrt{\varepsilon'_2}/\lambda_0$ and $\eta_2 = \eta_0/\sqrt{\varepsilon'_2}$.

The amplitude E_0^i is imposed by the source responsible for generating the incident wave, and therefore is assumed known. Our goal is to relate E_0^r and E_0^t to E_0^i. We do so by applying EM boundary conditions for the total electric and magnetic fields at $z = 0$. These boundary conditions state that the tangential component of the total electric field is always continuous across a boundary between two contiguous media, and in the absence of current sources at the boundary, the same is true for the total magnetic field. In the present case, the electric and magnetic fields of the incident, reflected, and transmitted waves are all tangential to the boundary. Application of boundary conditions at $z = 0$, which also is called ***phase matching***, leads to

$$E_0^r = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_0^i = \rho E_0^i, \qquad (2.87a)$$

$$E_0^t = \left(\frac{2\eta_2}{\eta_2 + \eta_1}\right) E_0^i = \tau E_0^i, \qquad (2.87b)$$

where

$$\rho = \frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \textbf{(normal incidence)}, \qquad (2.88a)$$

$$\tau = \frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_2 + \eta_1} \quad \textbf{(normal incidence)}. \qquad (2.88b)$$

The quantities ρ and τ are called the ***Fresnel reflection*** and ***transmission coefficients***. For lossless dielectric media, η_1 and η_2 are real; consequently, both ρ and τ are also real. From Eqs. (2.88a) and (2.88b), it is easy to show that ρ and τ are interrelated by

$$\tau = 1 + \rho \quad \textbf{(normal incidence)}. \qquad (2.89)$$

Using

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon'_1}},$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon'_2}},$$

in Eq. (2.88a) leads to

$$\rho = \frac{\sqrt{\varepsilon'_1} - \sqrt{\varepsilon'_2}}{\sqrt{\varepsilon'_1} + \sqrt{\varepsilon'_2}}. \qquad (2.90)$$

Table 2-3: Expressions for EM fields in lossless and lossy media under normal incidence.†

Lossless Media	Lossy Media
$\mathbf{E}^{\mathrm{i}} = \hat{\mathbf{y}} E_0^{\mathrm{i}} e^{jk_1 z}$, $\quad \mathbf{H}^{\mathrm{i}} = \hat{\mathbf{x}} \dfrac{E_0^{\mathrm{i}}}{\eta_1} e^{jk_1 z}$	$\mathbf{E}^{\mathrm{i}} = \hat{\mathbf{y}} E_0^{\mathrm{i}} e^{\gamma_1 z}$, $\quad \mathbf{H}^{\mathrm{i}} = \hat{\mathbf{x}} \dfrac{E_0^{\mathrm{i}}}{\eta_{c_1}} e^{\gamma_1 z}$
$\mathbf{E}^{\mathrm{r}} = \hat{\mathbf{y}} \rho E_0^{\mathrm{i}} e^{-jk_1 z}$, $\quad \mathbf{H}^{\mathrm{r}} = -\hat{\mathbf{x}} \rho \dfrac{E_0^{\mathrm{i}}}{\eta_1} e^{-jk_1 z}$	$\mathbf{E}^{\mathrm{r}} = \hat{\mathbf{y}} \rho E_0^{\mathrm{i}} e^{-\gamma_1 z}$, $\quad \mathbf{H}^{\mathrm{r}} = -\hat{\mathbf{x}} \rho \dfrac{E_0^{\mathrm{i}}}{\eta_{c_1}} e^{-\gamma_1 z}$
$\mathbf{E}^{\mathrm{t}} = \hat{\mathbf{y}} \tau E_0^{\mathrm{i}} e^{jk_2 z}$, $\quad \mathbf{H}^{\mathrm{t}} = \hat{\mathbf{x}} \tau \dfrac{E_0^{\mathrm{i}}}{\eta_2} e^{jk_2 z}$	$\mathbf{E}^{\mathrm{t}} = \hat{\mathbf{y}} \tau E_0^{\mathrm{i}} e^{\gamma_2 z}$, $\quad \mathbf{H}^{\mathrm{t}} = \hat{\mathbf{x}} \tau \dfrac{E_0^{\mathrm{i}}}{\eta_{c_2}} e^{\gamma_2 z}$
Phase matching: $(\mathbf{E}^{\mathrm{i}} + \mathbf{E}^{\mathrm{r}})\big\|_{z=0} = \mathbf{E}^{\mathrm{t}}\big\|_{z=0}$	Phase matching: $(\mathbf{E}^{\mathrm{i}} + \mathbf{E}^{\mathrm{r}})\big\|_{z=0} = \mathbf{E}^{\mathrm{t}}\big\|_{z=0}$
$(\mathbf{H}^{\mathrm{i}} + \mathbf{H}^{\mathrm{r}})\big\|_{z=0} = \mathbf{H}^{\mathrm{t}}\big\|_{z=0}$	$(\mathbf{H}^{\mathrm{i}} + \mathbf{H}^{\mathrm{r}})\big\|_{z=0} = \mathbf{H}^{\mathrm{t}}\big\|_{z=0}$
$\rho = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, $\quad \tau = 1 + \rho$	$\rho = \dfrac{\eta_{c_2} - \eta_{c_1}}{\eta_{c_2} + \eta_{c_1}}$, $\quad \tau = 1 + \rho$
$k_1 = \dfrac{2\pi}{\lambda_0} \sqrt{\varepsilon_1'}$, $\quad k_2 = \dfrac{2\pi}{\lambda_0} \sqrt{\varepsilon_2'}$	$\gamma_1 = \alpha_1 + j\beta_1$, $\quad \gamma_2 = \alpha_2 + j\beta_2$
$\eta_1 = \dfrac{\eta_0}{\sqrt{\varepsilon_1'}}$, $\quad \eta_2 = \dfrac{\eta_0}{\sqrt{\varepsilon_2'}}$	$\eta_{c_1} = \dfrac{\eta_0}{\sqrt{\varepsilon_1}}$, $\quad \eta_{c_2} = \dfrac{\eta_0}{\sqrt{\varepsilon_2}}$
Notes: $\varepsilon_1 = \varepsilon_1' - j\varepsilon_1''$; $\varepsilon_2 = \varepsilon_2' - j\varepsilon_2''$.	

Complete expressions for the fields and power densities in media 1 and 2 are available in the left-hand side of Table 2-3.

Using Eq. (2.71), the net average power density flowing in medium 1 is

$$\begin{aligned} \mathcal{S}_1(z) &= \tfrac{1}{2}\mathfrak{Re}[\mathbf{E}_1(z) \times \mathbf{H}_1^*(z)] \\ &= \tfrac{1}{2}\mathfrak{Re}\Bigg[\hat{\mathbf{y}} E_0^{\mathrm{i}}(e^{jk_1 z} + \rho e^{-jk_1 z}) \\ &\qquad \times \hat{\mathbf{x}} \frac{E_0^{\mathrm{i}*}}{\eta_1}(e^{-jk_1 z} - \rho^* e^{jk_1 z})\Bigg], \end{aligned}$$

which leads to

$$\mathcal{S}_1(z) = -\hat{\mathbf{z}} \frac{|E_0^{\mathrm{i}}|^2}{2\eta_1}(1 - |\rho|^2). \tag{2.91}$$

The first and second terms inside the bracket in Eq. (2.91) represent the average power density of the incident and reflected waves, respectively. Thus,

$$\mathcal{S}_1 = \mathcal{S}^{\mathrm{i}} + \mathcal{S}^{\mathrm{r}}, \tag{2.92a}$$

with

$$\mathcal{S}^{\mathrm{i}} = -\hat{\mathbf{z}} \frac{|E_0^{\mathrm{i}}|^2}{2\eta_1}, \tag{2.92b}$$

$$\mathcal{S}^{\mathrm{r}} = \hat{\mathbf{z}} |\rho|^2 \frac{|E_0^{\mathrm{i}}|^2}{2\eta_1} = \hat{\mathbf{z}} |\rho|^2 \mathcal{S}^{\mathrm{i}}. \tag{2.92c}$$

Even though ρ is purely real when both media are lossless dielectrics, we chose to treat it as complex, thereby providing in Eq. (2.92c) an expression that is also valid when medium 2 is conducting.

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† Computer Code 2.3.

The average power density of the transmitted wave in medium 2 is

$$\begin{aligned} \mathcal{S}^{\mathrm{t}}(z) = \mathcal{S}_2(z) &= \tfrac{1}{2}\mathfrak{Re}[\mathbf{E}_2(z) \times \mathbf{H}_2^*(z)] \\ &= \tfrac{1}{2}\mathfrak{Re}\left[\hat{\mathbf{y}}\tau E_0^{\mathrm{i}} e^{jk_2 z} \times \hat{\mathbf{x}}\tau^* \frac{E_0^{\mathrm{i}*}}{\eta_2} e^{-jk_2 z}\right] \\ &= -\hat{\mathbf{z}}|\tau|^2 \frac{|E_0^{\mathrm{i}}|^2}{2\eta_2}. \end{aligned} \tag{2.93}$$

Through the use of Eqs. (2.88a) and (2.88b), it can be easily shown that for lossless media

$$\frac{\tau^2}{\eta_2} = \frac{1-\rho^2}{\eta_1} \qquad \textbf{(lossless media)}, \tag{2.94}$$

which leads to

$$\mathcal{S}_1 = \mathcal{S}_2.$$

This result is expected from considerations of power conservation.

2-6.2 Boundary between Lossy Media

In Section 2-6.1 we considered a plane wave in a lossless medium incident normally on a planar boundary of another lossless medium. We now generalize our expressions to lossy media. In a medium with constitutive parameters $(\varepsilon', \mu, \sigma)$, the propagation constant $\gamma = \alpha + j\beta$ and the intrinsic impedance η_c are both complex. General expressions for α, β, and η_c are given in Table 2-1. If media 1 and 2 have constitutive parameters $(\varepsilon_1', \mu_0, \sigma_1)$ and $(\varepsilon_2', \mu_0, \sigma_2)$, the expressions for the electric and magnetic fields in media 1 and 2 can be obtained from those for the lossless case listed in the left-hand section of Table 2-3 by replacing jk with γ and η with η_c. The results are listed in the right-hand section of Table 2-3.

2-7 Wave Reflection and Transmission at Oblique Incidence

In the preceding section we examined reflection and transmission of plane waves that are normally incident upon a planar interface between two different media. We now consider the oblique-incidence case depicted in Fig. 2-14, and for simplicity we start by assuming all media to be lossless, and later we extend our results to lossy media. The $z = 0$ plane forms the boundary between media 1 and 2 with constitutive parameters (ε_1', μ_0) and (ε_2', μ_0), respectively. The two lines in Fig. 2-14 with direction $\hat{\mathbf{k}}_\mathrm{i}$ represent rays drawn normal to the wavefront of the incident wave, and those along directions $\hat{\mathbf{k}}_\mathrm{r}$ and $\hat{\mathbf{k}}_\mathrm{t}$ are similarly associated with the reflected and transmitted waves. The ***angles of incidence***, ***reflection***, and ***transmission*** (or ***refraction***), defined with respect to the normal to the boundary (the z axis), are θ_1, θ_1', and θ_2, respectively. These three

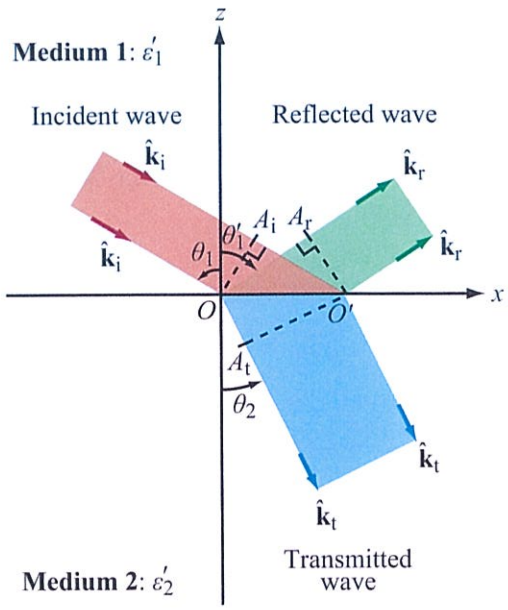


Figure 2-14: Wave reflection and refraction at a planar boundary between different media.

angles are interrelated by ***Snell's laws***:

$$\theta_1 = \theta_1' \quad \textbf{(Snell's law of reflection)},$$

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{u_{p_2}}{u_{p_1}} = \sqrt{\frac{\varepsilon_1'}{\varepsilon_2'}} \tag{2.95}$$

(Snell's law of refraction),

where u_{p_1} and u_{p_2} are the phase velocities in media 1 and 2, respectively.

▶ ***Snell's law of reflection*** states that the angle of reflection equals the angle of incidence, and ***Snell's law of refraction*** provides a relation between $\sin\theta_2$ and $\sin\theta_1$ in terms of the ratio of the phase velocities. ◀

The ***index of refraction*** of a medium, n, is defined as the ratio of the phase velocity in free space (i.e., the speed of light c) to the phase velocity in the medium. Thus,

$$n = \frac{c}{u_p} = \sqrt{\frac{\mu_0\varepsilon_0\varepsilon'}{\mu_0\varepsilon_0}} = \sqrt{\varepsilon'}\,. \tag{2.96}$$

In view of Eq. (2.96), Eq. (2.95) may be rewritten as

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2}\,. \tag{2.97}$$

Usually, materials with higher densities have higher permittivities. Air, with $\varepsilon' = 1$, has an index of refraction $n_0 = 1$. Since for nonmagnetic materials $n = \sqrt{\varepsilon'}$, *a material is often referred to as more dense than another material if it has a greater index of refraction.*

At normal incidence ($\theta_1 = 0$), Eq. (2.97) gives $\theta_2 = 0$, as expected. At oblique incidence $\theta_2 < \theta_1$ when $n_2 > n_1$ and $\theta_2 > \theta_1$ when $n_2 < n_1$.

▶ If a wave is incident on a more dense medium [Fig. 2-15(a)], the transmitted wave refracts inwardly (toward the z axis), and the opposite is true if a wave is incident on a less dense medium [Fig. 2-15(b)]. ◀

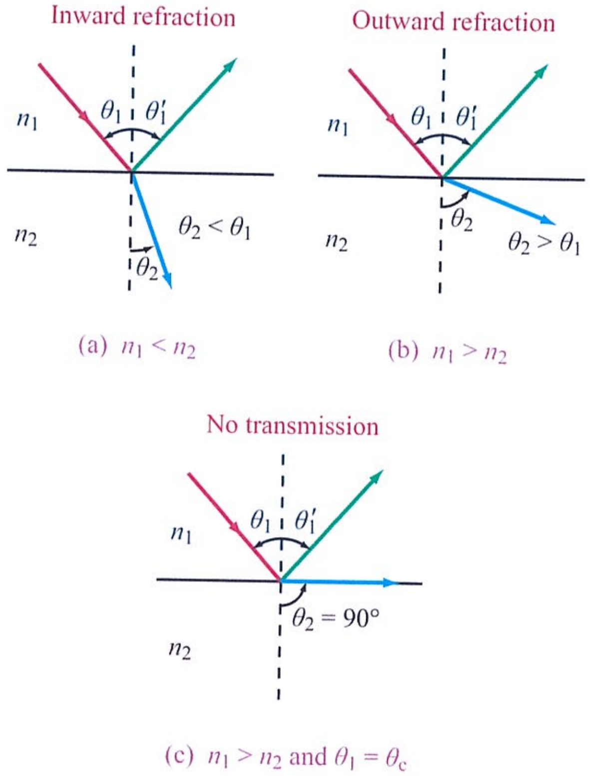


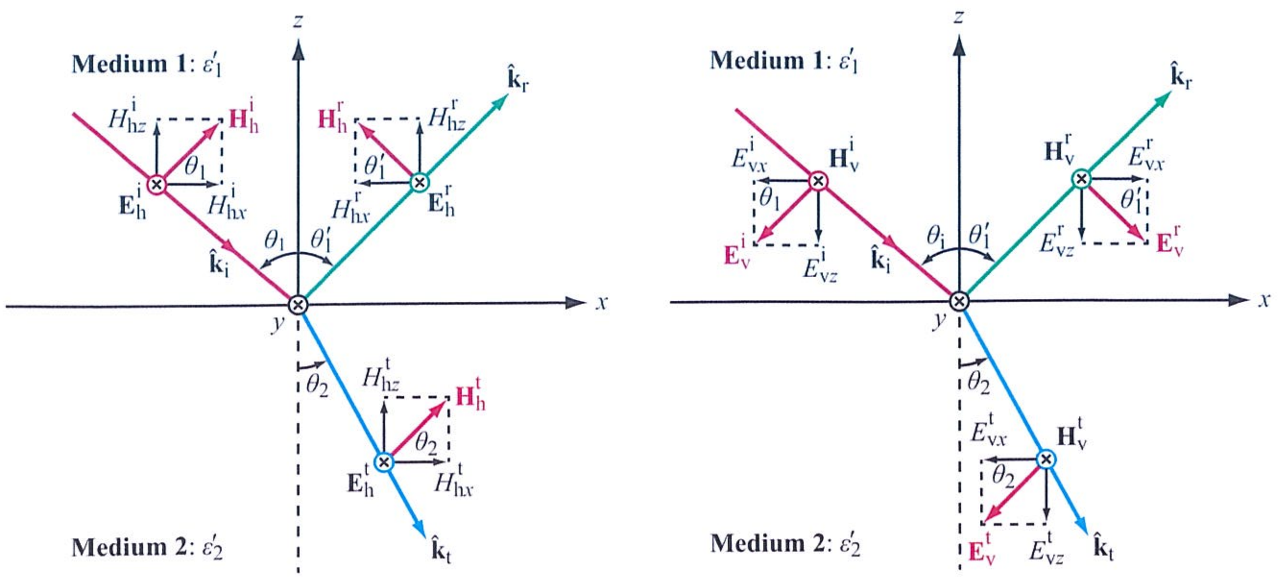
Figure 2-15: Snell's laws state that $\theta_1' = \theta_1$ and $\sin\theta_2 = (n_1/n_2)\sin\theta_1$. Refraction is (a) inward if $n_1 < n_2$ and (b) outward if $n_1 > n_2$; and (c) the refraction angle is 90° if $n_1 > n_2$ and θ_1 is equal to or greater than the critical angle $\theta_c = \sin^{-1}(n_2/n_1)$.

A case of particular interest is when $\theta_2 = \pi/2$, as shown in Fig. 2-15(c); in this case, the refracted wave flows along the surface and no energy is transmitted into medium 2. The value of the angle of incidence θ_1 corresponding to $\theta_2 = \pi/2$ is called the ***critical angle*** θ_c and is obtained from Eq. (2.97) as

$$\sin\theta_c = \frac{n_2}{n_1}\sin\theta_2\Big|_{\theta_2=\pi/2} = \frac{n_2}{n_1} \tag{2.98a}$$

$$= \sqrt{\frac{\varepsilon_2'}{\varepsilon_1'}}\,. \tag{2.98b}$$

(critical angle)



(a) Horizontal polarization (b) Vertical polarization

Figure 2-16: The plane of incidence is the plane containing the direction of wave travel, $\hat{\mathbf{k}}_i$, and the surface normal to the boundary. In the present case the plane of incidence containing $\hat{\mathbf{k}}_i$ and $\hat{\mathbf{z}}$ coincides with the plane of the paper. A wave is (a) perpendicularly polarized (also called ***horizontally polarized***) when its electric field vector is perpendicular to the plane of incidence and (b) parallel polarized (also called ***vertically polarized***) when its electric field vector lies in the plane of incidence.

If θ_1 exceeds θ_c, the incident wave is totally reflected, and the refracted wave becomes a nonuniform ***surface wave*** that travels along the boundary between the two media. This wave behavior is called ***total internal reflection***.

For normal incidence, the reflection and transmission coefficients ρ and τ at a boundary between two media are independent of the polarization of the incident wave, as both the electric and magnetic fields of a normally incident plane wave are tangential to the boundary regardless of the wave polarization. This is not the case for obliquely incident waves traveling at an angle $\theta_1 \neq 0$ with respect to the normal to the interface. In what follows, the ***plane of incidence*** *is defined as the plane containing the normal to the boundary and the direction of propagation of the incident wave.* A wave of arbitrary polarization may be described as the superposition of two orthogonally polarized waves, one with its electric field parallel to the plane of incidence (***parallel polarization***) and the other with its electric field perpendicular to the plane of incidence (***perpendicular polarization***). These two polarization configurations are shown in Fig. 2-16, in which the plane of incidence is coincident with the x–z plane.

▶ In remote sensing, the polarization with **E** perpendicular to the plane of incidence is also called ***horizontal polarization*** because **E** is parallel to Earth's surface, and that with **E** parallel to the plane of incidence is called ***vertical polarization*** because in this case it is the magnetic field that is parallel to Earth's surface. ◀

For the general case of a wave with an arbitrary polarization, it is common practice to decompose the incident wave $(\mathbf{E}^{\rm i}, \mathbf{H}^{\rm i})$ into a horizontally polarized component $(\mathbf{E}^{\rm i}_{\rm h}, \mathbf{H}^{\rm i}_{\rm h})$ and a vertically polarized component $(\mathbf{E}^{\rm i}_{\rm v}, \mathbf{H}^{\rm i}_{\rm v})$. Then, after determining the reflected waves $(\mathbf{E}^{\rm r}_{\rm h}, \mathbf{H}^{\rm r}_{\rm h})$ and $(\mathbf{E}^{\rm r}_{\rm v}, \mathbf{H}^{\rm r}_{\rm v})$ due to the two incident components, the reflected waves are added together to give the total reflected wave $(\mathbf{E}^{\rm r}, \mathbf{H}^{\rm r})$ corresponding to the original incident wave. A similar process can be used to determine the total transmitted wave $(\mathbf{E}^{\rm t}, \mathbf{H}^{\rm t})$.

2-7.1 Horizontal Polarization—Lossless Media

In the normal-incidence configuration depicted in Fig. 2-13, $\mathbf{E}^{\rm i}$, $\mathbf{H}^{\rm i}$, and $\hat{\mathbf{k}}_{\rm i}$ of the incident wave pointed along $\hat{\mathbf{y}}$, $\hat{\mathbf{x}}$, and $-\hat{\mathbf{z}}$, respectively. For the horizontally polarized incident wave depicted in Fig. 2-16(a), $\mathbf{E}^{\rm i}_{\rm h}$ continues to be along $\hat{\mathbf{y}}$, but $\mathbf{H}^{\rm i}_{\rm h}$ and $\hat{\mathbf{k}}_{\rm i}$ point along new directions. Accordingly, the electric and magnetic fields of the incident plane wave are given by

$$\mathbf{E}^{\rm i}_{\rm h} = \hat{\mathbf{y}} E^{\rm i}_{\rm h0} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)},$$

$$\mathbf{H}^{\rm i}_{\rm h} = (\hat{\mathbf{x}}\cos\theta_1 + \hat{\mathbf{z}}\sin\theta_1)\,\frac{E^{\rm i}_{h0}}{\eta_1}\, e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}.$$

Similar expressions can be written for the reflected and transmitted wave (and are given in Table 2-4). Boundary conditions require that the tangential component of the total electric field in medium 1 (i.e., the sum of $\mathbf{E}^{\rm i}_{\rm h}$ and $\mathbf{E}^{\rm r}_{\rm h}$) be equal to the tangential component of $\mathbf{E}^{\rm t}$ at the boundary ($z = 0$). In this case all three electric fields are along $\hat{\mathbf{y}}$ and, therefore, tangential to the x–y boundary. A similar boundary condition applies to the $\hat{\mathbf{x}}$ components of the magnetic fields. Application of these boundary conditions leads to the following expressions for the reflection and transmission coefficients in the horizontal polarization case:

$$\rho_{\rm h} = \frac{E^{\rm r}_{\rm h0}}{E^{\rm i}_{\rm h0}} = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}, \tag{2.99a}$$

$$\tau_{\rm h} = \frac{E^{\rm t}_{\rm h0}}{E^{\rm i}_{\rm h0}} = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}. \tag{2.99b}$$

These two coefficients, which formally are known as the ***Fresnel reflection and transmission coefficients for horizontal polarization***, are related by

$$\tau_{\rm h} = 1 + \rho_{\rm h}. \tag{2.100}$$

If medium 2 is a perfect conductor ($\eta_2 = 0$), Eqs. (2.99a) and (2.99b) reduce to $\rho_{\rm h} = -1$ and $\tau_{\rm h} = 0$, respectively, which means that the incident wave is totally reflected by the conducting medium.

In view of Eq. (2.95), the expression for $\rho_{\rm h}$ can be written as

$$\rho_{\rm h} = \frac{\cos\theta_1 - \sqrt{(\varepsilon'_2/\varepsilon'_1) - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{(\varepsilon'_2/\varepsilon'_1) - \sin^2\theta_1}}. \tag{2.101}$$

Since $(\varepsilon_2/\varepsilon_1) = (n_2/n_1)^2$, this expression can also be written in terms of the indices of refraction n_1 and n_2.

2-7.2 Vertical Polarization

Due to the dual nature of the electromagnetic fields, the expressions for the vertical polarization case can be obtained from the horizontal polarization case by replacing $\mathbf{E}$ with $\mathbf{H}$ and $\mathbf{H}$ with $-\mathbf{E}$. The process leads to the expressions given in Table 2-4. Of particular note are the reflection and transmission coefficients,

$$\rho_{\rm v} = \frac{E^{\rm r}_{\rm v0}}{E^{\rm i}_{\rm v0}} = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1},$$
$$= \frac{\left(\dfrac{\varepsilon'_2}{\varepsilon'_1} - \sin^2\theta_1\right)^{1/2} - \left(\dfrac{\varepsilon'_2}{\varepsilon'_1}\right)\cos\theta_1}{\left(\dfrac{\varepsilon'_2}{\varepsilon'_1} - \sin^2\theta_1\right)^{1/2} + \left(\dfrac{\varepsilon'_2}{\varepsilon'_1}\right)\cos\theta_1}, \tag{2.102a}$$

and

$$\tau_{\rm v} = \frac{E^{\rm t}_{\rm v0}}{E^{\rm i}_{\rm v0}} = (1 + \rho_{\rm v})\,\frac{\cos\theta_1}{\cos\theta_2}. \tag{2.102b}$$

To illustrate the angular variations of the magnitudes of $\rho_{\rm h}$ and $\rho_{\rm v}$, Fig. 2-17 shows plots for waves incident in air onto three different types of dielectric surfaces: dry

Table 2-4: Expressions for EM fields in lossless and lossy* media under normal incidence.†

Horizontal Polarization	Vertical Polarization
$\mathbf{E}_h^i = \hat{\mathbf{y}} E_{h0}^i e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$	$\mathbf{E}_v^i = (-\hat{\mathbf{x}}\cos\theta_1 - \hat{\mathbf{z}}\sin\theta_1) E_{v0}^i e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$
$\mathbf{H}_h^i = (\hat{\mathbf{x}}\cos\theta_1 + \hat{\mathbf{z}}\sin\theta_1)\dfrac{E_{h0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$	$\mathbf{H}_v^i = \hat{\mathbf{y}}\dfrac{E_{v0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$
$\mathbf{E}_h^r = \hat{\mathbf{y}} \rho_h E_{h0}^i e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$	$\mathbf{E}_v^r = (\hat{\mathbf{x}}\cos\theta_1 - \hat{\mathbf{z}}\sin\theta_1)\rho_v E_{v0}^i e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$
$\mathbf{H}_h^r = (-\hat{\mathbf{x}}\cos\theta_1 + \hat{\mathbf{z}}\sin\theta_1)\rho_h \dfrac{E_{h0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$	$\mathbf{H}_v^r = \hat{\mathbf{y}}\rho_v \dfrac{E_{v0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$
$\mathbf{E}_h^t = \hat{\mathbf{y}} \tau_h E_{h0}^i e^{-jk_2(x\sin\theta_2 - z\cos\theta_2)}$	$\mathbf{E}_v^t = (-\hat{\mathbf{x}}\cos\theta_2 - \hat{\mathbf{z}}\sin\theta_2)\tau_v E_{v0}^i e^{-jk_2(x\sin\theta_2 - z\cos\theta_2)}$
$\mathbf{H}_h^t = (\hat{\mathbf{x}}\cos\theta_2 + \hat{\mathbf{z}}\sin\theta_2)\tau_h \dfrac{E_{h0}^i}{\eta_2} e^{-jk_2(x\sin\theta_2 - z\cos\theta_2)}$	$\mathbf{H}_v^t = \hat{\mathbf{y}}\tau_v \dfrac{E_{v0}^i}{\eta_1} e^{-jk_2(x\sin\theta_2 - z\cos\theta_2)}$
$\rho_h = \dfrac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2} = \dfrac{\cos\theta_1 - \sqrt{(\varepsilon_2'/\varepsilon_1') - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{(\varepsilon_2'/\varepsilon_1') - \sin^2\theta_1}}$	$\rho_v = \dfrac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1} = \dfrac{\left[\dfrac{\varepsilon_2'}{\varepsilon_1'} - \sin^2\theta_1\right]^{1/2} - \left(\dfrac{\varepsilon_2'}{\varepsilon_1'}\right)\cos\theta_1}{\left[\dfrac{\varepsilon_2'}{\varepsilon_1'} - \sin^2\theta_1\right]^{1/2} + \left(\dfrac{\varepsilon_2'}{\varepsilon_1'}\right)\cos\theta_1}$
$\tau_h = 1 + \rho_h$	$\tau_v = (1 + \rho_v)\dfrac{\cos\theta_1}{\cos\theta_2}$
$k_1 = \dfrac{2\pi}{\lambda_0}\sqrt{\varepsilon_1}$ $\qquad \eta_1 = \dfrac{\eta_0}{\sqrt{\varepsilon_1}}$	$\eta_2 = \dfrac{\eta_0}{\sqrt{\varepsilon_2}}$
$k_2 = \dfrac{2\pi}{\lambda}\sqrt{\varepsilon_2}$ $\qquad k_2\sin\theta_2 = k_1\sin\theta_1$ $\cos\theta_2 = \left[1 - \left(\dfrac{k_1}{k_2}\sin\theta_1\right)^2\right]^{1/2}$	
*Notes: (1) Lossless medium: $\varepsilon = \varepsilon'$; (2) lossy medium: $\varepsilon = \varepsilon' - j\varepsilon''$ and jk should be replaced with $\gamma = j(2\pi\sqrt{\varepsilon' - j\varepsilon''})/\lambda_0$.	

soil ($\varepsilon_2' = 3$), wet soil ($\varepsilon_2' = 25$), and water ($\varepsilon_2' = 81$). For each of the surfaces, (a) $\rho_h = \rho_v$ at normal incidence ($\theta_1 = 0$), as expected, (b) $|\rho_h| = |\rho_v| = 1$ at ***grazing incidence*** ($\theta_1 = 90°$), and (c) ρ_v goes to zero at an angle called the ***Brewster angle***. For nonmagnetic materials, the Brewster angle exists only for vertical polarization, and its value depends on the ratio ($\varepsilon_2'/\varepsilon_1'$).

▶ At the Brewster angle, the vertically polarized component of the incident wave is totally transmitted into medium 2. ◀

From Eq. (2.102a), $\rho_v = 0$ when

$$\eta_1\cos\theta_1 = \eta_2\cos\theta_2. \qquad (2.103)$$

Combining Eq. (2.103) with Snell's law for nonmagnetic media, namely $\sin\theta_2 = (\eta_2/\eta_1)\sin\theta_1$, and then denot-

†**Computer Code 2.3.**

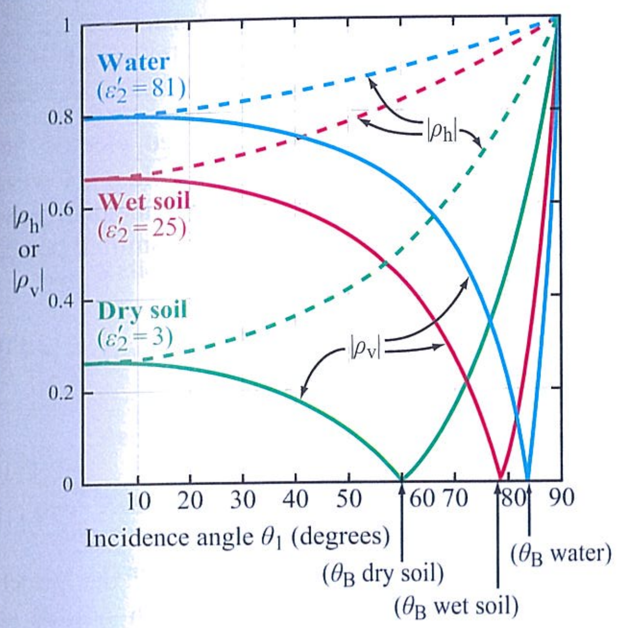


Figure 2-17: Plots for $|\rho_h|$ and $|\rho_v|$ as a function of θ_1 for a dry soil surface, a wet-soil surface, and a water surface. For each surface, $|\rho_v| = 0$ at the Brewster angle.

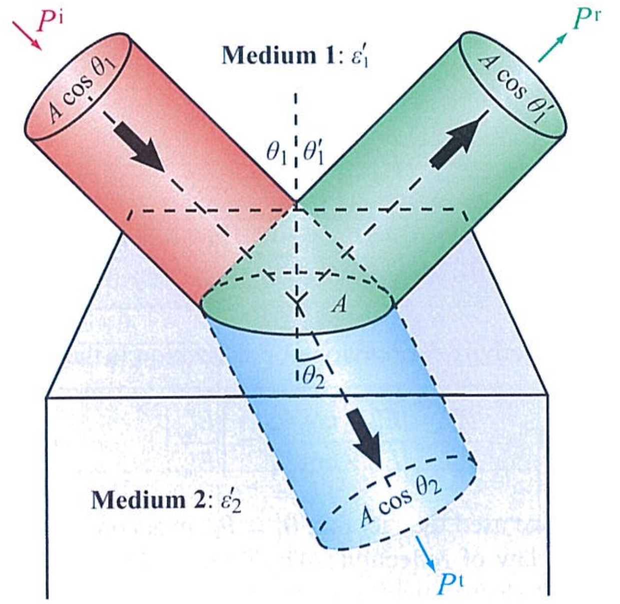


Figure 2-18: Reflection and transmission of an incident circular beam illuminating a spot of size A on the interface.

ing θ_1 as θ_B, leads to

$$\theta_B = \sin^{-1}\sqrt{\frac{1}{1 + (\varepsilon_1'/\varepsilon_2')}} = \tan^{-1}\sqrt{\frac{\varepsilon_2'}{\varepsilon_1'}}. \quad (2.104)$$

The Brewster angle is also called the ***polarizing angle***. This is because, if a wave composed of both horizontal and vertical polarization components is incident upon a nonmagnetic surface at the Brewster angle θ_B, the vertically polarized component is totally transmitted into the second medium, and only the horizontally polarized component is reflected by the surface.

2-8 Reflectivity and Transmissivity

The reflection and transmission coefficients are ratios of the reflected and transmitted electric field amplitudes to the amplitude of the incident electric field. We now examine power ratios, starting with the horizontal polarization case. Figure 2-18 shows a circular beam of electromagnetic energy incident upon the boundary between two contiguous, lossless media. The area of the spot illuminated by the beam is A, and the incident, reflected, and transmitted beams have electric-field amplitudes E^i_{h0}, E^r_{h0}, and E^t_{h0}, respectively. The average magnitudes of the power densities carried by the incident, reflected, and transmitted beams are

$$\mathcal{S}^i_h = \frac{|E^i_{h0}|^2}{2\eta_1}, \qquad \mathcal{S}^r_h = \frac{|E^r_{h0}|^2}{2\eta_1}, \qquad \mathcal{S}^t_h = \frac{|E^t_{h0}|^2}{2\eta_2},$$

where η_1 and η_2 are the intrinsic impedances of media 1 and 2, respectively. The cross-sectional areas of the incident, reflected, and transmitted beams are

$$A_i = A\cos\theta_1, \qquad A_r = A\cos\theta_1', \qquad A_t = A\cos\theta_2,$$

and the corresponding average powers carried by the beams are

$$P^i_h = \mathcal{S}^i_h A_i = \frac{|E^i_{h0}|^2}{2\eta_1} A\cos\theta_1, \quad (2.105a)$$

$$P_h^r = S_h^r A_r = \frac{|E_{h0}^r|^2}{2\eta_1} A\cos\theta_1', \tag{2.105b}$$

$$P_h^t = S_h^t A_t = \frac{|E_{h0}^t|^2}{2\eta_2} A\cos\theta_2. \tag{2.105c}$$

▶ The ***reflectivity*** Γ (also called ***reflectance*** in optics) is defined as the ratio of the reflected power to the incident power. ◀

The reflectivity for horizontal polarization is then

$$\Gamma^h = \frac{P_h^r}{P_h^i} = \frac{|E_{h0}^r|^2\cos\theta_1'}{|E_{h0}^i|^2\cos\theta_1} = \left|\frac{E_{h0}^r}{E_{h0}^i}\right|^2, \tag{2.106}$$

where we used the fact that $\theta_1' = \theta_1$, in accordance with Snell's law of reflection. The ratio of the reflected to incident electric field amplitudes, $|E_{h0}^r/E_{h0}^i|$, is equal to the magnitude of the reflection coefficient ρ_h. Hence,

$$\Gamma^h = |\rho_h|^2, \tag{2.107}$$

and, similarly, for vertical polarization

$$\Gamma^v = \frac{P_v^r}{P_v^i} = |\rho_v|^2. \tag{2.108}$$

▶ The ***transmissivity*** $\mathbb{T}$ (or ***transmittance*** in optics) is defined as the ratio of the transmitted power to incident power. ◀

That is,

$$\mathbb{T}^h = \frac{P_h^t}{P_h^i} = \frac{|E_{h0}^t|^2}{|E_{h0}^i|^2}\,\frac{\eta_1}{\eta_2}\,\frac{A\cos\theta_2}{A\cos\theta_1} = |\tau_h|^2\left(\frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1}\right), \tag{2.109a}$$

$$\mathbb{T}^v = \frac{P_v^t}{P_v^i} = |\tau_v|^2\left(\frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1}\right). \tag{2.109b}$$

▶ The incident, reflected, and transmitted waves have to obey the law of conservation of power. ◀

In fact, in many cases the transmitted electric field is larger than the incident electric field. However, conservation of power requires that the incident power equals the sum of the reflected and transmitted powers. That is, for horizontal polarization,

$$P_h^i = P_h^r + P_h^t, \tag{2.110}$$

which leads to

$$\Gamma^h + \mathbb{T}^h = 1, \tag{2.111a}$$

$$\Gamma^v + \mathbb{T}^v = 1, \tag{2.111b}$$

Table 2-5 provides a summary of the general expressions for ρ, τ, Γ, and $\mathbb{T}$ for both normal and oblique incidence.

2-9 Oblique Incidence onto a Lossy Medium

Consider a plane EM wave incident in medium 1 upon a planar boundary between medium 1 and medium 2, with medium 1 being a lossless dielectric and medium 2 a lossy medium. Water is highly lossy at microwave frequencies (Section 4-2), so lakes and oceans are perfect examples of lossy media. Medium 1 is characterized by constitutive parameters (ε_1' and μ_0) and medium 2 is characterized by ($\varepsilon_2 = \varepsilon_2' - j\varepsilon_2''$ and μ_0). In turn,

$$k_1 = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_1'}, \qquad \eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_1'}}, \qquad \textbf{(medium 1)} \tag{2.112a}$$

$$\gamma_2 = \alpha_2 + j\beta_2, \qquad \eta_{c_2} = \frac{\eta_0}{\sqrt{\varepsilon_2}}, \qquad \textbf{(medium 2)} \tag{2.112b}$$

with α_2 and β_2 as given by the expressions in Table 2-1.

Formally, the only modifications we need to make to the expressions given earlier in Table 2-4 (for the

Table 2-5: Expressions for ρ, τ, Γ, and $\mathbb{T}$ for wave incidence from a lossless medium with intrinsic impedance η_1 onto a second lossless medium with intrinsic impedance η_2. Angles θ_1 and θ_2 are the angles of incidence and transmission, respectively.[†]

Property	Normal Incidence $\theta_1 = \theta_2 = 0$	Horizontal Polarization	Vertical Polarization
Reflection coefficient	$\rho = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$	$\rho_h = \dfrac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$	$\rho_v = \dfrac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1}$
Transmission coefficient	$\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}$	$\tau_h = \dfrac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$	$\tau_v = \dfrac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1}$
Relation of ρ to τ	$\tau = 1 + \rho$	$\tau_h = 1 + \rho_h$	$\tau_v = (1 + \rho_v) \dfrac{\cos\theta_1}{\cos\theta_2}$
Reflectivity	$\Gamma = \lvert\rho\rvert^2$	$\Gamma^h = \lvert\rho_h\rvert^2$	$\Gamma^v = \lvert\rho_v\rvert^2$
Transmissivity	$\mathbb{T} = \lvert\tau\rvert^2 \left(\dfrac{\eta_1}{\eta_2}\right)$	$\mathbb{T}^h = \lvert\tau_h\rvert^2 \dfrac{\eta_1 \cos\theta_2}{\eta_2 \cos\theta_1}$	$\mathbb{T}^v = \lvert\tau_v\rvert^2 \dfrac{\eta_1 \cos\theta_2}{\eta_2 \cos\theta_1}$
Relation of Γ to $\mathbb{T}$	$\mathbb{T} = 1 - \Gamma$	$\mathbb{T}^h = 1 - \Gamma^h$	$\mathbb{T}^v = 1 - \Gamma^v$
Notes: $\sin\theta_2 = \sqrt{\varepsilon_1'/\varepsilon_2'} \sin\theta_1$; $\eta_1 = \eta_0/\sqrt{\varepsilon_1'}$; $\eta_2 = \eta_0/\sqrt{\varepsilon_2'}$.			

case where both media are lossless) are to replace η_2 with η_{c_2} and jk_2 with γ_2. The new expressions are listed in Table 2-6. For the incident and reflected fields in medium 1, the expressions remain unchanged, but those for the transmitted fields in the lossy medium 2 now involve $\gamma_2 \sin\theta_2$ and $\gamma_2 \cos\theta_2$. Snell's law (for both lossless and lossy media) requires the tangential components of the phase factors (which in the present case are the x components) to match at the boundary. That is,

$$\gamma_2 \sin\theta_2 = \gamma_1 \sin\theta_1 = jk_1 \sin\theta_1, \tag{2.113}$$

where we have replaced γ_1 with jk_1 because medium 1 is lossless. We still need an expression for $\cos\theta_2$ in terms of θ_1 and the constitutive parameters of the two media. To that end, we use Eq. (2.113) to obtain

$$\cos\theta_2 = [1 - \sin^2\theta_2]^{1/2}$$

$$= \left[1 - \left(\frac{jk_1}{\gamma_2} \sin\theta_1\right)^2\right]^{1/2}$$

$$= \left[1 - \left(\frac{jk_1}{\alpha_2 + j\beta_2} \sin\theta_1\right)^2\right]^{1/2}. \tag{2.114}$$

▶ Clearly $\cos\theta_2$ is a complex quantity, which means that θ_2 is no longer a real angle in the traditional sense. ◀

Nevertheless, for the purpose of computing ρ, τ, and the electric and magnetic fields in the transmitted medium, the expression given by Eq. (2.114) is all that is needed. Figure 2-19 displays angular plots of $|\rho_v|$ and $|\rho_h|$ for incidence in air upon a material with $\varepsilon_2 = \varepsilon_2' - j\varepsilon_2''$. The plots contrast three surfaces, all with $\varepsilon_2' = 50$, but with very different values for the loss factor ε_2''.

As noted in connection with Eq. (2.114), θ_2 is a complex quantity whose sine and cosine functions

[†] Computer Code 2.3.

Table 2-6: Oblique incidence in a lossless medium onto a lossy medium.†

Horizontal Polarization	Vertical Polarization
$\mathbf{E}_\mathrm{h}^\mathrm{i} = \hat{\mathbf{y}} E_\mathrm{h0}^\mathrm{i} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$	$\mathbf{E}_\mathrm{v}^\mathrm{i} = (-\hat{\mathbf{x}}\cos\theta_1 - \hat{\mathbf{z}}\sin\theta_1) E_\mathrm{v0}^\mathrm{i} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$
$\mathbf{H}_\mathrm{h}^\mathrm{i} = (\hat{\mathbf{x}}\cos\theta_1 + \hat{\mathbf{z}}\sin\theta_1) \dfrac{E_\mathrm{h0}^\mathrm{i}}{\eta_1} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$	$\mathbf{H}_\mathrm{v}^\mathrm{i} = \hat{\mathbf{y}} \dfrac{E_\mathrm{v0}^\mathrm{i}}{\eta_1} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$
$\mathbf{E}_\mathrm{h}^\mathrm{r} = \hat{\mathbf{y}} \rho_\mathrm{h} E_\mathrm{h0}^\mathrm{i} e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$	$\mathbf{E}_\mathrm{v}^\mathrm{r} = (\hat{\mathbf{x}}\cos\theta_1 - \hat{\mathbf{z}}\sin\theta_1) \rho_\mathrm{v} E_\mathrm{v0}^\mathrm{i} e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$
$\mathbf{H}_\mathrm{h}^\mathrm{r} = (-\hat{\mathbf{x}}\cos\theta_1 + \hat{\mathbf{z}}\sin\theta_1) \rho_\mathrm{h} \dfrac{E_\mathrm{h0}^\mathrm{i}}{\eta_1} e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$	$\mathbf{H}_\mathrm{v}^\mathrm{r} = \hat{\mathbf{y}} \rho_\mathrm{v} \dfrac{E_\mathrm{v0}^\mathrm{i}}{\eta_1} e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$
$\mathbf{E}_\mathrm{h}^\mathrm{t} = \hat{\mathbf{y}} \tau_\mathrm{h} E_\mathrm{h0}^\mathrm{i} e^{-\gamma_2(x\sin\theta_2 - z\cos\theta_2)}$	$\mathbf{E}_\mathrm{v}^\mathrm{t} = (-\hat{\mathbf{x}}\cos\theta_2 - \hat{\mathbf{z}}\sin\theta_2) \tau_\mathrm{v} E_\mathrm{v0}^\mathrm{i} e^{-\gamma_2(x\sin\theta_2 - z\cos\theta_2)}$
$\mathbf{H}_\mathrm{h}^\mathrm{t} = (\hat{\mathbf{x}}\cos\theta_2 + \hat{\mathbf{z}}\sin\theta_2) \tau_\mathrm{h} \dfrac{E_\mathrm{h0}^\mathrm{i}}{\eta_2} e^{-\gamma_2(x\sin\theta_2 - z\cos\theta_2)}$	$\mathbf{H}_\mathrm{v}^\mathrm{t} = \hat{\mathbf{y}} \tau_\mathrm{v} \dfrac{E_\mathrm{v0}^\mathrm{i}}{\eta_1} e^{-\gamma_2(x\sin\theta_2 - z\cos\theta_2)}$
$\rho_\mathrm{h} = \dfrac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}$	$\rho_\mathrm{v} = \dfrac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1}$
$\tau_\mathrm{h} = 1 + \rho_\mathrm{h}$	$\tau_\mathrm{v} = (1 + \rho_\mathrm{v}) \dfrac{\cos\theta_1}{\cos\theta_2}$

Notes:

$k_1 = \dfrac{2\pi}{\lambda_0}\sqrt{\varepsilon_1'}$ $\eta_1 = \dfrac{\eta_0}{\sqrt{\varepsilon_1'}}$ $\eta_2 = \dfrac{\eta_0}{\sqrt{\varepsilon_2}}$

$\gamma_2 = \alpha_2 + j\beta_2 = j\dfrac{2\pi}{\lambda_0}\sqrt{\varepsilon_2}$ $\gamma_2\sin\theta_2 = k_1\sin\theta_1$ $\varepsilon_2 = \varepsilon_2' - j\varepsilon_2''$

$$\cos\theta_2 = \left[1 - \left(\frac{jk_1}{\gamma_2}\sin\theta_1\right)^2\right]^{1/2} = \left[1 - \left(\frac{\varepsilon_1'}{\varepsilon_2' - j\varepsilon_2''}\right)\sin^2\theta_1\right]^{1/2}$$

provide the necessary phase matching at the interface between the two media. The ***real angle*** of transmission χ_2 is given by (Stratton, 1941):

$$\chi_2 = \tan^{-1}\left[\frac{\sqrt{2}\,k_1\sin\theta_1}{[(p^2+q^2)^{1/2}+q]^{1/2}}\right],$$

where

$$p = 2\alpha_2\beta_2$$

and

$$q = \beta_2^2 - \alpha_2^2 - k_1^2\sin^2\theta_1 .$$

If medium 2 is lossless ($\alpha_2 = 0$), the expression for χ_2 reduces to $\chi_2 = \theta_2$.

†Computer Code 2.3.

2-10 Oblique Incidence onto a Two-Layer Composite

The formulations presented in the preceding section allow us to compute the reflection by and transmission through an ocean surface when observed by a downward-looking airborne or spaceborne microwave sensor. For a nadir-looking ($\theta_1 = 0$) altimeter, the reflected pulse measured by its receiver is proportional to the reflection coefficient ρ of the surface. Side-looking radars measure a quantity called the ***backscattering coefficient*** of the surface, σ^0, which is related to ρ_v and ρ_h of the surface. A radiometer measures the emissivity of the surface, which is related to $(1 - |\rho_\mathrm{v}|^2)$

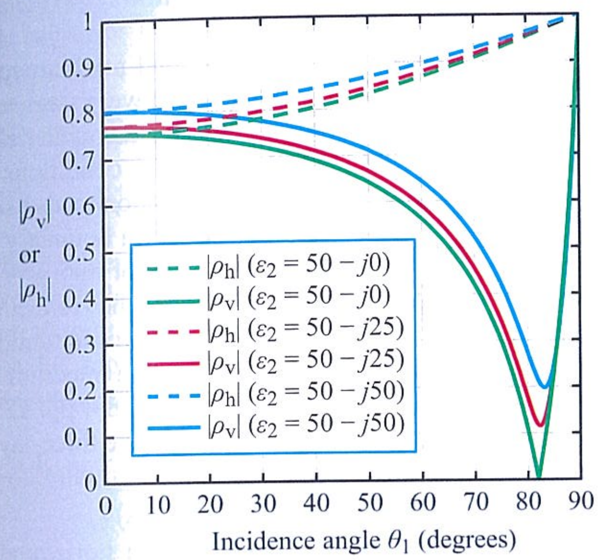


Figure 2-19: Angular plots of $|\rho_v|$ and $|\rho_h|$ for incidence in air upon a material with $\varepsilon_2 = \varepsilon_2' - j\varepsilon_2''$.

for vertical polarization and to $(1 - |\rho_h|^2)$ for horizontal polarization. So for all of these sensor configurations, it is important to be able to model the reflection coefficient of lossless, low-loss, and lossy media.

A specific scenario of interest is when a lake or ocean water surface is covered by a uniform layer of ice or oil (due to spillage by oil tankers). A similar scenario occurs when a soil surface is covered by a layer of snow. At microwave frequencies, oil is a low-loss material and so is lake ice. Sea ice may or may not behave like a low-loss medium depending on its type and the specific microwave frequency used by the sensor. Snow is a low-loss medium when dry and lossy when wet. To compute the reflectivity of such two-layer composites, we model the problem as a three-layer configuration:

Medium 1: Air
Medium 2: Intervening uniform layer of thickness d (ice, oil, or snow)
Medium 3: Water (lossy)

Before we proceed with the development of an appropriate mathematical model for ρ_v and ρ_h, however, we should examine the physics of the multilayer reflection process. Figure 2-20 depicts a uniform plane wave represented by a ray incident at angle θ_1 in medium 1. In addition to the reflection at the boundary between media 1 and 2, part of the incident ray crosses the boundary and propagates down to the second boundary that separates medium 2 from medium 3, thereupon part of it is reflected back up towards the upper boundary. The upward-propagating ray in medium 2 gets both partly reflected by the upper boundary and partly transmitted across into the top medium. The multiple reflection process continues indefinitely, with each successive reflection carrying less energy than its predecessor.

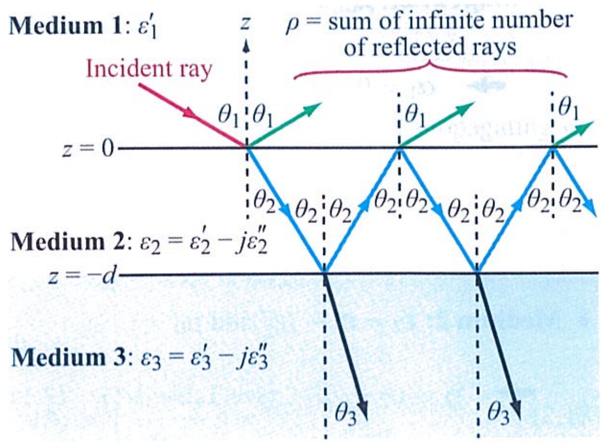


Figure 2-20: Multiple reflections in a two-layer composite.

Under steady state conditions, the reflection coefficient in medium 1 is equal to the sum of all of the electric fields of the upward directed rays in medium 1, divided by the amplitude of the electric field of the incident wave.

2-10.1 Input Parameters

(a) Constitutive Parameters (Fig. 2-20)

- **Medium 1:** ε_1', μ_0, and $\sigma_1 = 0$

$$\alpha_1 = 0, \qquad \gamma_1 = jk_1 = j\,\frac{2\pi}{\lambda_0}\sqrt{\varepsilon_1'}\,, \tag{2.115a}$$

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_1'}} \tag{2.115b}$$

- **Medium 2:** $\varepsilon_2 = \varepsilon_2' - j\varepsilon_2''$ and μ_0

$$\gamma_2 = \alpha_2 + j\beta_2 \quad \text{(see Table 2-1)}, \tag{2.116a}$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_2}}\,. \tag{2.116b}$$

If medium 2 is low-loss (i.e., $\varepsilon_2''/\varepsilon_2' \ll 1$), then

$$\alpha_2 \approx \frac{\pi}{\lambda_0}\,\frac{\varepsilon_2''}{\sqrt{\varepsilon_2'}}\,, \qquad \beta_2 \approx \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_2'}\,. \tag{2.116c}$$

- **Medium 3:** $\varepsilon_3 = \varepsilon_3' - j\varepsilon_3''$ and μ_0

$$\gamma_3 = \alpha_3 + j\beta_3 \quad \text{(see Table 2-1)}, \tag{2.117}$$

$$\eta_3 = \frac{\eta_0}{\sqrt{\varepsilon_3}}\,. \tag{2.118}$$

(b) Snell's Law Phase-Matching Condition

- $\gamma_1 \sin\theta_1 = \gamma_2 \sin\theta_2 = \gamma_3 \sin\theta_3$ (2.119a)

$$\cos\theta_2 = \left[1 - \left(\frac{\gamma_1}{\gamma_2}\sin\theta_1\right)^2\right]^{1/2} \tag{2.119b}$$

$$\approx \left[1 - \left(\frac{\beta_1}{\beta_2}\sin\theta_1\right)^2\right]^{1/2} \tag{2.119c}$$

if medium 2 is low loss

$$\cos\theta_3 = \left[1 - \left(\frac{\gamma_1}{\gamma_3}\sin\theta_1\right)^2\right]^{1/2}. \tag{2.120}$$

2-10.2 Propagation Matrix Method

Reflection and transmission by a layered medium consisting of N layers with known constitutive parameters can be computed by defining electric and magnetic fields in each layer and then applying boundary conditions at all $(N-1)$ boundaries. Each such application lends itself into a matrix that relates the fields above a boundary to those below it. Successive multiplication of the $(N-1)$ matrices can lead to computing both the reflection coefficient in medium 1 of the N-layer configuration, as well as the transmission coefficient in medium N (Tsang et al., 1985, pp. 25–31; Balanis, 1989, pp. 229–236). We now apply the matrix method to the three-layer configuration shown in Fig. 2-21, and we initially do so for horizontal polarization.

From Table 2-6, the electric field of a horizontally polarized wave incident in medium 1 (Fig. 2-21) is given by the general form

$$\mathbf{E}_1^- = \hat{\mathbf{y}} A_1 e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}, \tag{2.121}$$

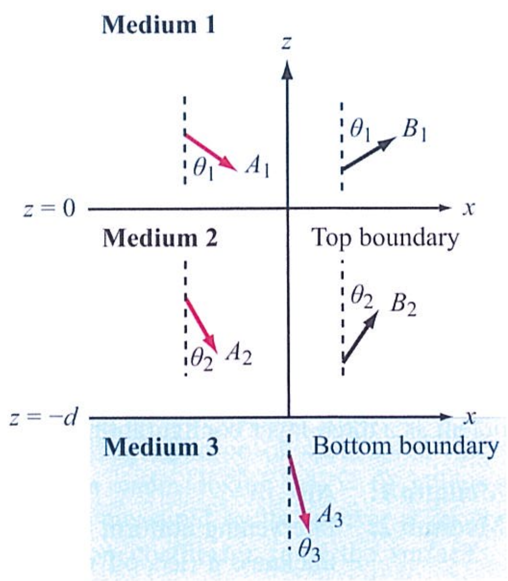


Figure 2-21: Electric field amplitudes of downward and upward propagating waves in media 1, 2 and 3.

where A_1 is the amplitude of $\mathbf{E}_1^-$ at $x = 0$ and $z = 0$, the subscript 1 denotes that $\mathbf{E}_1^-$ is in medium 1, and the superscript $(-)$ denotes that it belongs to a downward-propagating wave. Similarly, the electric field of the upward-propagating wave in medium 1 is

$$\mathbf{E}_1^+ = \hat{\mathbf{y}} B_1 e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}, \tag{2.122}$$

where B_1 represents the amplitude of the sum total of the electric fields of all multiple-reflection components as they exist in medium 1, and the superscript $(+)$ denotes that $\mathbf{E}_1^+$ represents an upward propagating wave. The total electric field in medium 1 is

$$\begin{aligned}\mathbf{E}_1 &= \mathbf{E}_1^- + \mathbf{E}_1^+ \\ &= \hat{\mathbf{y}}(A_1 e^{jk_1 z\cos\theta_1} + B_1 e^{-jk_1 z\cos\theta_1}) e^{-jk_1 x\sin\theta_1}.\end{aligned} \tag{2.123}$$

Application of Eq. (2.34a) leads to the associated magnetic fields $\mathbf{H}_1^-$ and $\mathbf{H}_1^+$ and to their sum:

$$\mathbf{H}_1 = \hat{\mathbf{x}} H_{1x} + \hat{\mathbf{z}} H_{1z}, \tag{2.124}$$

with

$$H_{1x} = \frac{\cos\theta_1}{\eta_1}(A_1 e^{jk_1 z\cos\theta_1} - B_1 e^{-jk_1 z\cos\theta_1}) e^{-jk_1 x\sin\theta_1}, \tag{2.125a}$$

$$H_{1z} = \frac{\sin\theta_1}{\eta_1}(A_1 e^{jk_1 z\cos\theta_1} + B_1 e^{-jk_1 z\cos\theta_1}) e^{-jk_1 x\sin\theta_1}. \tag{2.125b}$$

Extending the formulation to medium 2 entails replacing A_1 and B_1 with A_2 and B_2, respectively, jk_1 with γ_2, and η_1 with η_{c_2}. The process leads to

$$\begin{aligned}\mathbf{E}_2 &= \mathbf{E}_2^- + \mathbf{E}_2^+ \\ &= \hat{\mathbf{y}}(A_2 e^{\gamma_2 z\cos\theta_2} + B_2 e^{-\gamma_2 z\cos\theta_2}) e^{-\gamma_2 x\sin\theta_2},\end{aligned} \tag{2.126}$$

$$\mathbf{H}_2 = \hat{\mathbf{x}} H_{2x} + \hat{\mathbf{z}} H_{2z}, \tag{2.127}$$

with

$$H_{2x} = \frac{\cos\theta_2}{\eta_{c_2}}(A_2 e^{\gamma_2 z\cos\theta_2} - B_2 e^{-\gamma_2 z\cos\theta_2}) e^{-\gamma_2 x\sin\theta_2}, \tag{2.128a}$$

$$H_{2z} = \frac{\sin\theta_2}{\eta_{c_2}}(A_2 e^{\gamma_2 z\cos\theta_2} + B_2 e^{-\gamma_2 z\cos\theta_2}) e^{-\gamma_2 x\sin\theta_2}. \tag{2.128b}$$

Medium 3 contains only a downward-propagating wave. Hence,

$$\mathbf{E}_3 = \hat{\mathbf{y}}(A_3 e^{\gamma_3 z\cos\theta_3}) e^{-\gamma_3 x\sin\theta_3}, \tag{2.129}$$

$$\mathbf{H}_3 = \hat{\mathbf{x}} H_{3x} + \hat{\mathbf{z}} H_{3z}, \tag{2.130}$$

with

$$H_{3x} = \left(\frac{\cos\theta_3}{\eta_{c_3}} A_3 e^{\gamma_3 z\cos\theta_3}\right) e^{-\gamma_3 x\sin\theta_3}, \tag{2.131a}$$

$$H_{3z} = \left(\frac{\sin\theta_3}{\eta_{c_3}} A_3 e^{\gamma_3 z\cos\theta_3}\right) e^{-\gamma_3 x\sin\theta_3}. \tag{2.131b}$$

In view of Eq. (2.119a) and keeping in mind that in the present case $\gamma_1 = jk_1$ because medium 1 is lossless, it follows that

$$\underbrace{e^{-jk_1 x\sin\theta_1}}_{\text{medium 1}} = \underbrace{e^{-\gamma_2 x\sin\theta_2}}_{\text{medium 2}} = \underbrace{e^{-\gamma_3 x\sin\theta_3}}_{\text{medium 3}}. \tag{2.132}$$

Hence, the x component of the phase in all of the expressions given by Eqs. (2.123) through (2.131) are identical (which is another way to state Snell's law).

At the top boundary (at $z = 0$), the tangential components of $\mathbf{E}$ and $\mathbf{H}$ must be continuous. Consequently,

$$\mathbf{E}_1|_{z=0} = \mathbf{E}_2|_{z=0},$$

which, upon equating Eq. (2.123) to Eq. (2.126) and setting $z = 0$, leads to

$$A_1 + B_1 = A_2 + B_2. \tag{2.133}$$

For the magnetic field, continuity at $z = 0$ applies to its x component. That is,

$$H_{1x}|_{z=0} = H_{2x}|_{z=0},$$

which leads to

$$\frac{\cos\theta_1}{\eta_1}(A_1 - B_1) = \frac{\cos\theta_2}{\eta_2}(A_2 - B_2). \tag{2.134}$$

Similarly, at the bottom boundary (at $z = -d$),

$$\mathbf{E}_2|_{z=-d} = \mathbf{E}_3|_{z=-d} \quad \text{and} \quad H_{2x}|_{z=-d} = H_{3x}|_{z=-d},$$

which leads to

$$(A_2 e^{-\gamma_2 d \cos\theta_2} + B_2 e^{\gamma_2 d \cos\theta_2}) = A_3 e^{-\gamma_3 d \cos\theta_3}, \quad (2.135)$$

$$\frac{\cos\theta_2}{\eta_2}(A_2 e^{-\gamma_2 d \cos\theta_2} - B_2 e^{\gamma_2 d \cos\theta_2}) = \frac{A_3 \cos\theta_3}{\eta_3} e^{-\gamma_3 d \cos\theta_3}. \quad (2.136)$$

The effective reflection coefficient of the combined two-layer structure (media 2 and 3) is given by

$$\rho = \frac{B_1}{A_1}. \quad (2.137)$$

Simultaneous solution of Eqs. (2.133) through (2.136) leads to

$$\rho = \frac{\rho_{12} + \rho_{23} e^{-2\gamma_2 d \cos\theta_2}}{1 + \rho_{12}\rho_{23} e^{-2\gamma_2 d \cos\theta_2}} \quad \textbf{(h or v polarization)},^{\dagger} \quad (2.138)$$

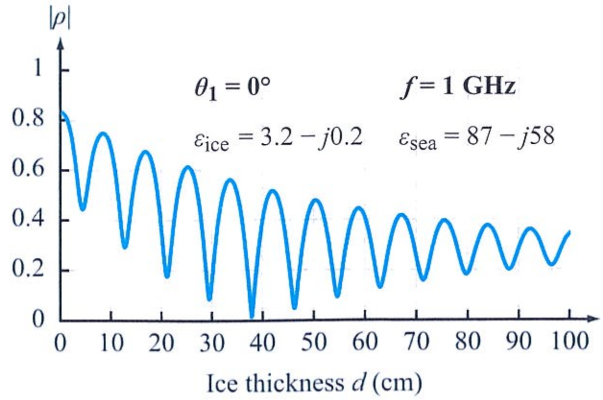
where ρ_{12} is the reflection coefficient at the top boundary for incidence in semi-infinite medium 1 upon semi-infinite medium 2 (i.e., as if it were the only boundary). Similarly, ρ_{23} is the reflection coefficient at the bottom boundary for incidence in medium 2 upon medium 3. For horizontal polarization, the expressions for ρ_{12} and ρ_{23} are given by

$$\left.\begin{aligned} \rho_{12} &= \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2} \\ \rho_{23} &= \frac{\eta_3 \cos\theta_2 - \eta_2 \cos\theta_3}{\eta_3 \cos\theta_2 + \eta_2 \cos\theta_3} \end{aligned}\right\} \quad \textbf{(h polarization)}. \quad (2.139)$$

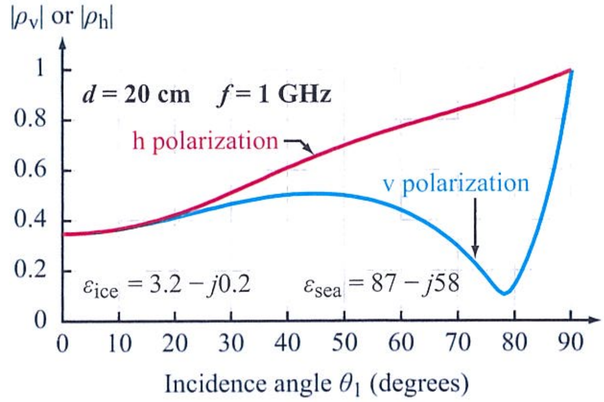
For the vertically polarized case:

$$\left.\begin{aligned} \rho_{12} &= \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} \\ \rho_{23} &= \frac{\eta_3 \cos\theta_3 - \eta_2 \cos\theta_2}{\eta_3 \cos\theta_3 + \eta_2 \cos\theta_2} \end{aligned}\right\} \quad \textbf{(v polarization)}. \quad (2.140)$$

Figure 2-22 displays two examples:



(a) $|\rho|$ versus d



(b) $|\rho_v|$ and $|\rho_h|$ versus θ

Figure 2-22: Plots of (a) $|\rho|$ as a function of ice-layer thickness d for normal incidence and (b) $|\rho_v|$ and $|\rho_h|$ as a function of incidence angle θ_1 for a 20 cm ice layer over the ocean surface.

(a) a plot of the magnitude of the normal-incidence reflection coefficient, $|\rho|$, as a function of d, where d is the thickness of an ice layer over the ocean surface, and

(b) plots of $|\rho_v|$ and $|\rho_h|$ as a function of incidence angle θ_1 for an ice layer of thickness $d = 20$ cm.

The oscillatory behavior exhibited by $|\rho|$ as a function of d in Fig. 2-22(a) has maxima and minima corresponding to, respectively, constructive and

† Computer Code 2.4.

destructive interference of the multiple reflections that occur in the ice layer.

2-10.3 Multiple Reflection Method

The expression for ρ given by Eq. (2.138) was obtained by applying boundary conditions at the upper and lower boundaries of the middle layer in Fig. 2-21. We can gain additional insight by deriving Eq. (2.138) a second time by tracking all the multiple reflections and transmissions that occur at the two boundaries. We demonstrate the process for h polarization, and we suppress the x component of the phase factor since, as noted earlier, it is the same for all electric and magnetic fields in all three media.

Figure 2-23 depicts the various reflection, transmission, and propagation mechanisms encountered by a wave incident in medium 1 upon medium 2 at incidence angle θ_1. They include:

$\rho_{12} =$ reflection coefficient at the boundary between media 1 and 2, for incidence in medium 1 at angle θ_1.

$\rho_{21} =$ reflection coefficient at the boundary between media 1 and 2, for incidence in medium 2 at angle θ_2. Note that $\rho_{21} = -\rho_{12}$.

$\tau_{12} =$ transmission coefficient from medium 1 to medium 2 when incidence is at angle θ_2. Note that $\tau_{12} = 1 + \rho_{12}$ for h polarization and $\tau_{12} = (1 + \rho_{12})\cos\theta_1/\cos\theta_2$ for v polarization.

$\tau_{21} =$ transmission coefficient from medium 2 to medium 1 when incidence is at angle θ_2. Note that $\tau_{21} = 1 + \rho_{21}$ for h polarization and $\tau_{21} = (1 + \rho_{21})\cos\theta_2/\cos\theta_1$ for v polarization.

$\mathcal{L} =$ $e^{-\gamma_2 d\cos\theta_2}$ = propagation factor in medium 2 between its top boundary and bottom boundary (or between its bottom boundary and top boundary) along angle θ_2.

$\rho_{23} =$ reflection coefficient at the boundary between media 2 and 3 for incidence in medium 2 at angle θ_2.

For an incident field with $E_0^{\mathrm{i}} = 1$ V/m, the first reflection in Fig. 2-24 contributes ρ_{12}, the second one is $\tau_{21}\rho_{23}\mathcal{L}^2\tau_{12}$, and so on. Summing up all of the upward-moving waves in medium 1 gives

$$\begin{aligned} \rho &= \rho_{12} + \tau_{21}\rho_{23}\mathcal{L}^2\tau_{12} + \tau_{21}\rho_{23}^2\rho_{21}\mathcal{L}^4\tau_{12} + \cdots \\ &= \rho_{12} + \tau_{21}\tau_{12}\rho_{23}\mathcal{L}^2(1 + x + x^2 + \cdots), \end{aligned} \qquad (2.141)$$

where $x = \rho_{21}\rho_{23}\mathcal{L}^2$. We note that the magnitudes of ρ_{21}, ρ_{23}, and $\mathcal{L}^2$ are all smaller than 1. Hence, $x < 1$. If we make the substitutions

$$\tau_{21} = 1 + \rho_{21} = 1 - \rho_{12},$$

$$\tau_{12} = 1 + \rho_{12},$$

and

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots,$$

Eq. (2.141) becomes

$$\rho = \rho_{12} + \frac{(1-\rho_{12})(1+\rho_{12})\rho_{23}\mathcal{L}^2}{1 - \rho_{21}\rho_{23}\mathcal{L}^2}. \qquad (2.142)$$

Upon making the substitutions $\rho_{21} = -\rho_{12}$ and $\mathcal{L}^2 = e^{-2\gamma_2 d\cos\theta_2}$ and simplifying the expression, we obtain

$$\rho = \frac{\rho_{12} + \rho_{23}e^{-2\gamma_2 d\cos\theta_2}}{1 + \rho_{12}\rho_{23}e^{-2\gamma_2 d\cos\theta_2}},^{\dagger} \qquad (2.143)$$

which is identical with the result obtained in the previous subsection in the form of Eq. (2.138). The reflectivity of the two-layer composite is $\Gamma = |\rho|^2$.

PROBLEMS

2.1 Determine (a) the direction of propagation, (b) phase velocity, (c) wavelength, (d) relative permittivity of the material, and (e) the electric field phasor of an EM wave propagating in a nonmagnetic material, given that the wave's magnetic field is

$$\mathbf{H} = \hat{\mathbf{x}}6\cos(10^9 t - 20y) \qquad \text{(mA/m)}.$$

2.2 Determine (a) the direction of propagation, (b) phase velocity, (c) wavelength, (d) relative

†Computer Code 2.4.

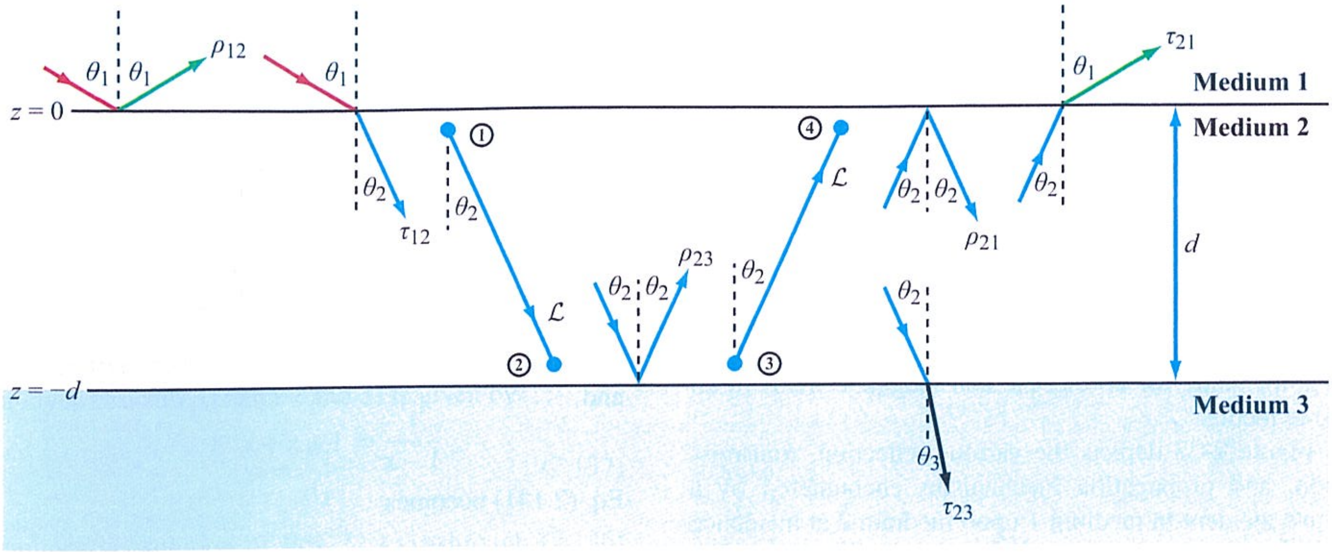


Figure 2-23: Reflection, transmission, and propagation mechanisms.

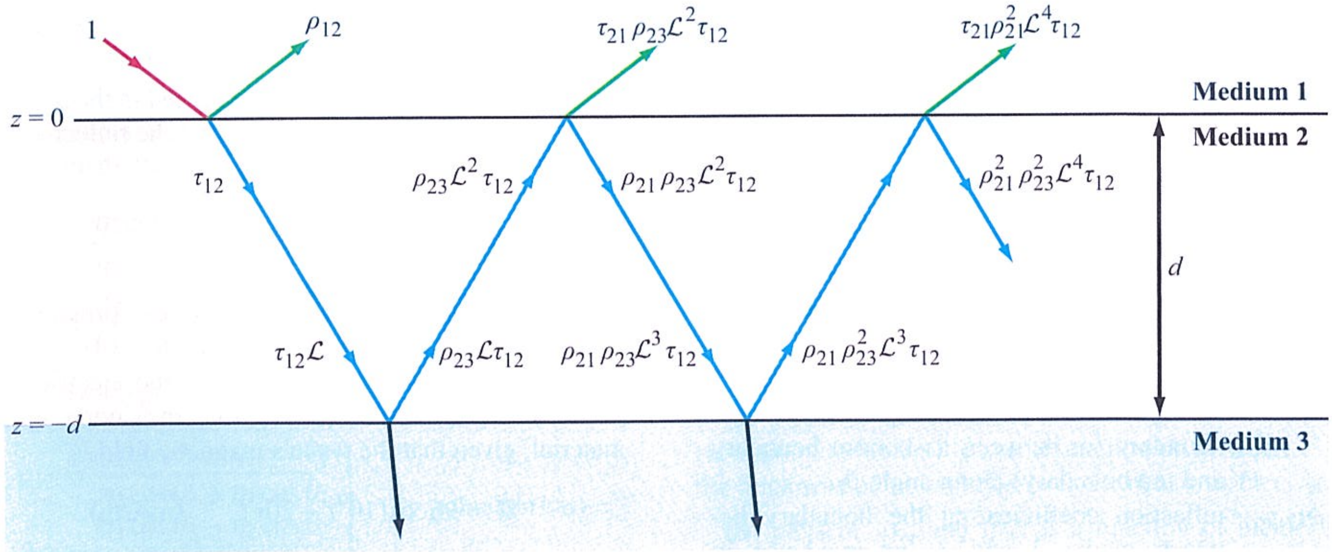


Figure 2-24: Multiple reflection process.

permittivity of the material, and (e) the electric field phasor of an EM wave propagating in a nonmagnetic material, given that the wave's magnetic field is

$$\mathbf{H} = \hat{\mathbf{z}} 60 \cos(10^7 t + 0.1x - 60^\circ) \qquad \text{(mA/m)}.$$

2.3 Provide general expressions for the electric and magnetic fields of a 10 GHz plane wave traveling in the $+x$ direction in a lossless nonmagnetic medium with relative permittivity $\varepsilon' = 4$. The electric field is polarized along the z direction, its peak value is 12 V/m, and its intensity is 8 V/m at $t = 0$ and $x = 0.5$ cm.

2.4 Provide general expressions for the electric and magnetic fields of a 30 GHz plane wave traveling in the $+z$ direction in a lossless nonmagnetic medium with relative permittivity $\varepsilon' = 16$. The electric field is polarized along the z direction, its peak value is 10 V/m, and its intensity is 6 V/m at $t = 0$ and $z = 2$ cm.

2.5 The electric field of a plane EM wave propagating in a nonmagnetic medium is given by

$$\mathbf{E} = \hat{\mathbf{x}} 6 \cos(2 \times 10^7 t - 0.4y) + \hat{\mathbf{z}} 3 \sin(2 \times 10^7 t - 0.4y) \qquad \text{(V/m)}.$$

Determine: (a) λ, (b) ε', and (c) $\mathbf{H}$.

2.6 The electric field of a plane EM wave propagating in a nonmagnetic medium is given by

$$\mathbf{E} = \hat{\mathbf{y}} 12 \sin(2 \times 10^7 t - 0.1x) + \hat{\mathbf{z}} 4 \cos(2 \times 10^7 t - 0.1x) \qquad \text{(V/m)}.$$

Determine: (a) λ, (b) ε', and (c) $\mathbf{H}$.

2.7 The electric field of a plane wave propagating in a nonmagnetic material with $\varepsilon' = 9$ is given by

$$\mathbf{E} = \hat{\mathbf{x}} 100 \cos(1.2\pi \times 10^{10} t + kz) \qquad \text{(V/m)}.$$

Determine: f, u_p, λ, k, η, and $\mathbf{H}$.

2.8 An LHC-polarized wave with a modulus of 2 (V/m) is traveling in free space in the negative z direction. Write the expression for the wave's electric field vector, given that the wavelength is 6 cm.

2.9 For a wave characterized by the electric field

$$\mathbf{E}(z,t) = \hat{\mathbf{x}} a_x \cos(\omega t - kz) + \hat{\mathbf{y}} a_y \cos(\omega t - kz + \delta)$$

identify the polarization state, determine the polarization angles (ψ, χ), and sketch the locus of $\mathbf{E}(0,t)$ for each of the following cases:

(a) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = 0$

(b) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = 180^\circ$

(c) $a_x = 3$ V/m, $a_y = 3$ V/m, and $\delta = 45^\circ$

(d) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = -135^\circ$

2.10 The electric field of a uniform plane wave propagating in free space is given by

$$\mathbf{E} = (\hat{\mathbf{x}} + j\hat{\mathbf{y}}) 60 e^{-j\pi z/6} \qquad \text{(V/m)}.$$

Specify the modulus and direction of the electric field intensity at the $z = 0$ plane at $t = 0$, 5, and 10 ns.

2.11 A linearly polarized plane wave of the form $\mathbf{E} = \hat{\mathbf{x}} a_x e^{-jkz}$ can be expressed as the sum of an RHC polarized wave with magnitude a_R, and an LHC polarized wave with magnitude a_L. Prove this statement by finding expressions for a_R and a_L in terms of a_x.

2.12 The electric field of an elliptically polarized plane wave is given by

$$\mathbf{E}(z,t) = [-\hat{\mathbf{x}} 20 \sin(\omega t - kz - 60^\circ) + \hat{\mathbf{y}} 30 \cos(\omega t - kz)] \qquad \text{(V/m)}.$$

Determine: (a) the polarization angles (ψ, χ), and (b) the direction of rotation.

2.13 Compare the polarization states of each of the following pairs of plane waves:

(a) Wave 1: $\mathbf{E}_1 = \hat{\mathbf{x}} 4 \cos(\omega t - kz) + \hat{\mathbf{y}} 4 \sin(\omega t - kz)$.
Wave 2: $\mathbf{E}_2 = \hat{\mathbf{x}} 4 \cos(\omega t + kz) + \hat{\mathbf{y}} 4 \sin(\omega t + kz)$.

(b) Wave 1: $\mathbf{E}_1 = \hat{\mathbf{x}} 4 \cos(\omega t - kz) - \hat{\mathbf{y}} 4 \sin(\omega t - kz)$.
Wave 2: $\mathbf{E}_2 = \hat{\mathbf{x}} 4 \cos(\omega t + kz) - \hat{\mathbf{y}} 4 \sin(\omega t + kz)$.

2.14 Plot the locus of $\mathbf{E}(0,t)$ for a plane wave with

$$\mathbf{E}(z,t) = \hat{\mathbf{x}} \sin(\omega t + kz) + \hat{\mathbf{y}} 2 \cos(\omega t + kz).$$

Determine the polarization state from your plot.

2.15 For each of the following combinations of parameters, determine if the material is a low-loss dielectric, a quasi conductor, or a good conductor, and then calculate α, β, λ, u_p, and η_c:

(a) Glass with $\mu = \mu_0$, $\varepsilon' = 5$, and $\sigma = 10^{-12}$ S/m at 10 GHz.

(b) Animal tissue with $\mu = \mu_0$, $\varepsilon' = 12$, and $\sigma = 0.3$ S/m at 100 MHz.

(c) Wood with $\mu = \mu_0$, $\varepsilon' = 3$, and $\sigma = 10^{-4}$ S/m at 1 kHz.

2.16 Dry soil is characterized by $\varepsilon' = 2.5$, $\mu = \mu_0$, and $\sigma = 10^{-4}$ (S/m). At each of the following frequencies, determine if dry soil may be considered a good conductor, a quasi conductor, or a low-loss dielectric, and then calculate α, β, λ, μ_p, and η: (a) 60 Hz, (b) 1 kHz, (c) 1 MHz, (d) 1 GHz.

2.17 In a medium characterized by $\varepsilon' = 9$, $\mu = \mu_0$, and $\sigma = 0.1$ S/m, determine the phase angle by which the magnetic field leads the electric field at (a) 1 MHz, (b) 1 GHz, and (c) 10 GHz.

2.18 Generate a plot for the skin depth δ_s versus frequency for seawater for the range from 1 kHz to 10 GHz (use log-log scales). The constitutive parameters of seawater are $\mu = \mu_0$, $\varepsilon' = 80$, and $\sigma = 4$ S/m.

2.19 Wet soil is characterized by $\mu = \mu_0$, $\varepsilon' = 9$, and $\sigma = 5 \times 10^{-4}$ S/m. Ignoring reflection at the air–soil boundary, if the amplitude of an incident wave is 10 V/m at the surface of a wet soil medium, at what depth will it be down to 1 mV/m if (a) $f = 1$ MHz, (b) 1 GHz, and (c) 10 GHz?

2.20 Based on wave attenuation and reflection measurements conducted at 1 MHz, it was determined that the intrinsic impedance of a certain medium is $28.1\angle 45^\circ$ (Ω) and the skin depth is 2 m. Determine: (a) the conductivity of the material, (b) the wavelength in the medium, and (c) the phase velocity.

2.21 The electric field of a plane wave propagating in a nonmagnetic medium is given by

$$\mathbf{E} = \hat{\mathbf{z}}\,25e^{-30x}\cos(2\pi \times 10^9 t - 40x) \qquad \text{(V/m)}.$$

Obtain the corresponding expressions for $\mathbf{H}(t)$ and $\mathbf{S}(t)$.

2.22 The electric field of a plane wave propagating in a nonmagnetic medium is given by

$$\mathbf{E} = \hat{\mathbf{x}}10e^{-15z}\cos(4\pi \times 10^9 t - 80z) + \hat{\mathbf{y}}20e^{-15z}\cos(4\pi \times 10^9 t - 80z + 30^\circ) \quad \text{(V/m)}.$$

Determine the average power density carried by the wave.

2.23 A wave traveling in a nonmagnetic medium with $\varepsilon' = 9$ is characterized by an electric field given by

$$\mathbf{E} = [\hat{\mathbf{y}}\,3\cos(\pi \times 10^7 t + kx) - \hat{\mathbf{z}}\,2\cos(\pi \times 10^7 t + kx)]e^{-0.2x} \qquad \text{(V/m)}.$$

Determine the direction of wave travel and average power density carried by the wave.

2.24 The electric-field phasor of a uniform plane wave traveling downward in water is given by

$$\mathbf{E} = \hat{\mathbf{x}}\,5e^{-0.2z}e^{-j0.2z} \qquad \text{(V/m)},$$

where $\hat{\mathbf{z}}$ is the downward direction and $z = 0$ is the water surface. If $\sigma = 4$ S/m, (a) obtain an expression for the average power density, and (b) determine the depth at which the power density has been reduced by 40 dB.

2.25 The amplitudes of an elliptically polarized plane wave traveling in a lossless, nonmagnetic medium with $\varepsilon' = 4$ are $H_{y0} = 3$ (mA/m) and $H_{z0} = 4$ (mA/m). Determine the average power flowing through an aperture in the y–z plane if its area is 20 m^2.

2.26 At microwave frequencies, the power density considered safe for human exposure is 1 (mW/cm^2). A radar radiates a wave with an electric field amplitude E that decays with distance as $E(R) = (3{,}000/R)$ (V/m), where R is the distance in meters. What is the radius of the unsafe region?

2.27 Consider the imaginary rectangular box shown in Fig. P2.27. A wave traveling in the medium has electric and magnetic fields

$$\mathbf{E} = \hat{\mathbf{x}}\,100e^{-20y}\cos(2\pi \times 10^9 t - 40y) \qquad \text{(V/m)},$$

$$\mathbf{H} = -\hat{\mathbf{z}}0.64e^{-20y}\cos(2\pi \times 10^9 t - 40y - 36.85°) \quad \text{(A/m)}.$$

The box has dimensions $a = 1$ cm, $b = 2$ cm, and $c = 0.5$ cm. Determine: (a) the net time-average power entering the box, (b) the time-average power exiting the box, and (c) the time-average power absorbed by the box.

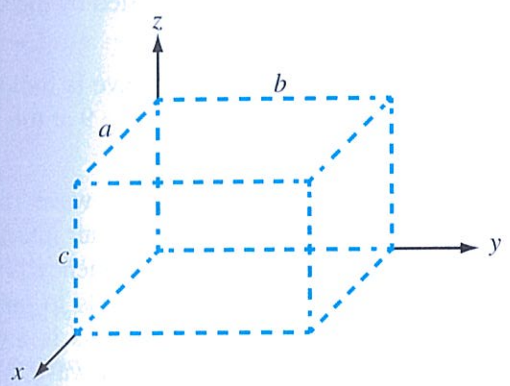


Figure P2.27: Imaginary rectangular box of Problem 2.27.

2.28 A team of scientists is designing a radar as a probe for measuring the depth of the ice layer over the Antarctic land mass. In order to measure a detectable echo due to the reflection by the ice–rock boundary, the thickness of the ice sheet should not exceed three skin depths. If $\varepsilon' = 3$ and $\varepsilon'' = 10^{-2}$ for ice and if the maximum anticipated ice thickness in the area under exploration is 1.2 km, what frequency range is usable with the radar?

2.29 A plane wave traveling in medium 1 with $\varepsilon'_1 = 2.25$ is normally incident upon medium 2 with $\varepsilon'_2 = 4$. Both media are made of nonmagnetic, nonconducting materials. If the electric field of the incident wave is given by

$$\mathbf{E}^{\mathrm{i}} = \hat{\mathbf{y}}8\cos(6\pi \times 10^9 t - 30\pi x) \qquad \text{(V/m)}.$$

(a) Obtain time-domain expressions for the electric and magnetic fields in each of the two media.

(b) Determine the average power densities of the incident, reflected and transmitted waves.

2.30 A plane wave traveling in a medium with $\varepsilon'_1 = 9$ is normally incident upon a second medium with $\varepsilon'_2 = 4$. Both media are made of nonmagnetic, nonconducting materials. If the magnetic field of the incident plane wave is given by

$$\mathbf{H}^{\mathrm{i}} = \hat{\mathbf{z}}2\cos(2\pi \times 10^9 t - ky) \qquad \text{(A/m)}.$$

(a) Obtain time-domain expressions for the electric and magnetic fields in each of the two media.

(b) Determine the average power densities of the incident, reflected, and transmitted waves.

2.31 A 200 MHz LHC-polarized plane wave with an electric field modulus of 5 V/m is normally incident in air upon a dielectric medium with $\varepsilon' = 4$, and occupies the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that the field is a positive maximum at $z = 0$ and $t = 0$.

(b) Calculate the reflection and transmission coefficients.

(c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

2.32 The three regions shown in Fig. P2.32 contain perfect dielectrics. For a wave in medium 1, incident normally upon the boundary at $z = -d$, what combination of ε'_2 and d produces no reflection? Express your answers in terms of ε'_1, ε'_3 and the oscillation frequency of the wave, f.

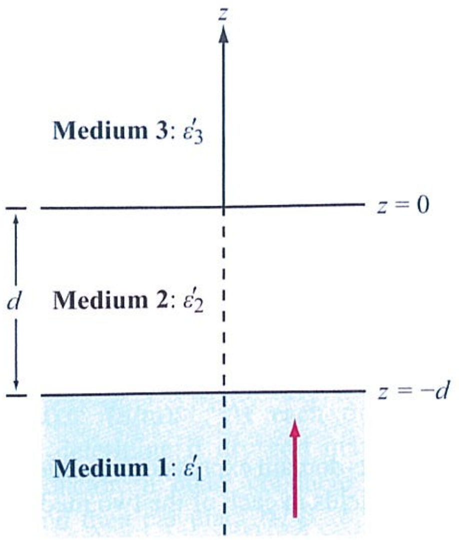


Figure P2.32: Dielectric layers for Problem 2.32.

2.33 A plane wave in air with

$$\mathbf{E}^{\mathrm{i}} = \hat{\mathbf{y}}\,20e^{-j(3x+4z)} \qquad \text{(V/m)}$$

is incident upon the planar surface of a dielectric material, with $\varepsilon' = 4$, occupying the half-space $z \geq 0$. Determine:

(a) The polarization of the incident wave.

(b) The angle of incidence.

(c) The time-domain expressions for the reflected electric and magnetic fields.

(d) The time-domain expressions for the transmitted electric and magnetic fields.

(e) The average power density carried by the wave in the dielectric medium.

2.34 Repeat Problem 2.33 for a wave in air with

$$\mathbf{H}^{\mathrm{i}} = \hat{\mathbf{y}}\,2\times 10^{-2}e^{-j(8x+6z)} \qquad \text{(A/m)}$$

incident upon the planar boundary of a dielectric medium ($z \geq 0$) with $\varepsilon' = 9$.

2.35 A plane wave in air with

$$\mathbf{E}^{\mathrm{i}} = (\hat{\mathbf{x}}9 - \hat{\mathbf{y}}4 - \hat{\mathbf{z}}6)e^{-j(2x+3z)} \qquad \text{(V/m)}$$

is incident upon the planar surface of a dielectric material, with $\varepsilon' = 2.25$, occupying the half-space $z \geq 0$. Determine:

(a) The incidence angle θ_1.

(b) The frequency of the wave.

(c) The field $\mathbf{E}^{\mathrm{r}}$ of the reflected wave.

(d) The field $\mathbf{E}^{\mathrm{t}}$ of the wave transmitted into the dielectric medium.

(e) The average power density carried by the wave into the dielectric medium.

2.36 A parallel-polarized plane wave is incident from air onto a dielectric medium with $\varepsilon' = 9$ at the Brewster angle. What is the refraction angle?

2.37 A perpendicularly polarized wave in air is obliquely incident upon a planar glass–air interface at an incidence angle of 30°. The wave frequency is 600 THz (1 THz $= 10^{12}$ Hz), which corresponds to green light, and the index of refraction of the glass is 1.6. If the electric field amplitude of the incident wave is 50 V/m, determine the following:

(a) The reflection and transmission coefficients.

(b) The instantaneous expressions for **E** and **H** in the glass medium.

2.38 A parallel-polarized beam of light in air with an electric field amplitude of 10 (V/m) is incident on polystyrene with $\mu = \mu_0$ and $\varepsilon' = 2.6$. If the incidence angle at the air–polystyrene planar boundary is 50°, determine the following:

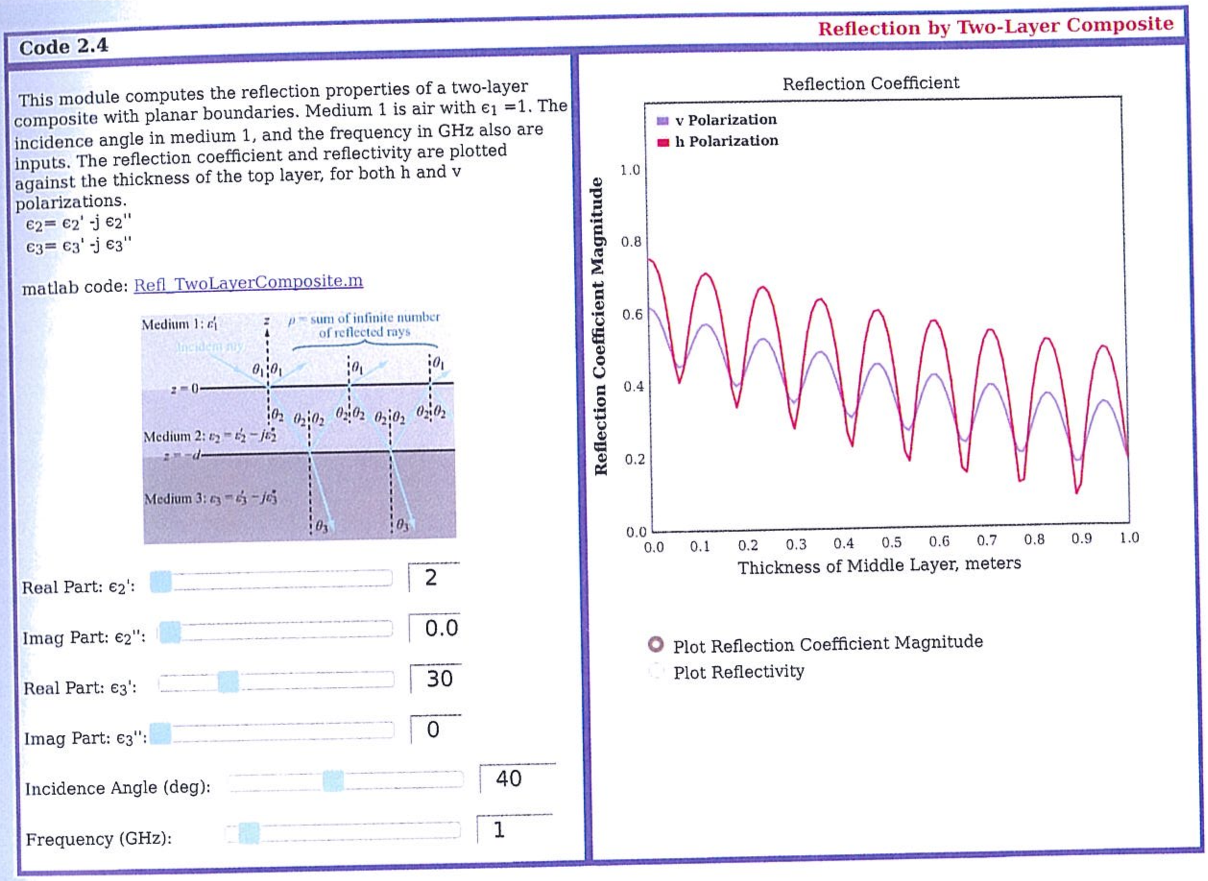
(a) The reflectivity and transmissivity.

(b) The power carried by the incident, reflected, and transmitted beams if the spot on the boundary illuminated by the incident beam is 1 m^2 in area.

2.39 A 50 MHz RHC-polarized plane wave with an electric field modulus of 30 V/m is normally incident in air upon a dielectric medium with $\varepsilon' = 9$ and occupying the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that the inclination angle is zero at $z = 0$ and $t = 0$.

(b) Calculate the reflection and transmission coefficients.



An example of one of the interactive modules available at the book website: mrs.eecs.umich.edu.

(c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

2.40 A horizontal sand layer of thickness d overlays a semi-infinite medium of wet soil. If $\varepsilon_{\text{sand}} = (3 - j0.1)$ and $\varepsilon_{\text{soil}} = (20 - j5)$, compute and plot the magnitude of the reflection coefficient, as a function of d from 0 to 1 m, at normal incidence at 2 GHz.

2.41 For the same sand-over-soil configuration of Problem 2.40, compute and plot the magnitude of the h-polarized reflection coefficient as a function of incidence angle from 0 to 90° for $d = 20$ cm. Keep f at 2 GHz.

2.42 At 20 GHz, the permittivity of ocean water at 20 °C is $\varepsilon_{\text{water}} = (36 - j30)$ and the permittivity of crude oil is $\varepsilon_{\text{oil}} = (2.1 - j0.1)$. The oil layer is of thickness d and overlays the water surface. Compute and plot the magnitude of the normal-incidence reflection coefficient as a function of d from 0 to 30 mm in 0.1 mm increments.

2.43 Repeat Problem 2.42 for horizontal polarization at an incidence angle of 50°.

2.44 Derive an expression for the transmission coefficient $\tau = A_3/A_1$ of the three-medium configuration treated in Section 2-10.2.